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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.E DEGREE END SEMESTER EXAMINATION- NOVEMBER 2010

SUB: ENGG. MATHEMATICS I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks

1A. Find the n^{th} derivatives of

i) $\frac{3x^2 - 5x - 1}{2x^3 - 3x^2 + 1}$ (ii) $xe^{2x}\sin^2 2x$

1B. A radius vector intersects the curve $r = ae^{\theta\cot\alpha}$ at consecutive points $P_0, P_1, \ldots, P_n \ldots$ If ρ_m and ρ_n denotes the radii of curvature at P_m and P_n, then show that $\frac{1}{m-n}\log\left(\frac{\rho_m}{\rho_n}\right)$ is independent of m and n for all $m \neq n$.

1C. Find the reflection of the point (1, 3, 4) through the plane $2x - y + z + 3 = 0$.

(4 + 3+ 3)

2A. Find the evolute of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

2B. Evaluate :

(i) $\int_0^{2a} x^4 \; 2ax - x^2 \;^{-1/2} dx$ (ii) $\int_0^{\infty} \frac{dx}{a^2 + x^2 \;^{n}}$

2C. If $y = \frac{d^n}{dx^n} \; x^2 - 1 \;^{n}$, then prove that

$(1 - x^2)y_2 - 2xy_1 + n(n+1)y = 0$. (3 + 4+ 3)

3A. Find the angle between the curves

$r^2 \sin 2\theta = 4$, and $r^2 = 16\sin 2\theta$

3B. Test the Nature of the following series

(i) $\frac{3}{4} + \frac{3.6}{4.7} + \frac{3.6.9}{4.7.10} + \ldots$

(ii) $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \ldots\ldots$

3C. Trace the following curve with explanation
$y(1-x^2)=x^2$

(3+ 4+ 3)

4A. State Cauchy's mean value theorem and verify it for
$f(x)=\sqrt{x}$ and $g(x)=\frac{1}{\sqrt{x}}$ in [a,b]

4B. Find the magnitude and equations of the line of shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-5}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$
Also find the points where it intersects the lines.

4C. Obtain the first three nonzero terms in the Maclaurin's series expansion of
$f(x)=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$.

(3+ 4 + 3)

5A. If $u = f(x^2+y^2+z^2)$ where $x = r\cos\theta\cos\phi$, $y = r\cos\theta\sin\phi$, $z = r\sin\theta$ find $\frac{\partial u}{\partial\theta}$ and $\frac{\partial u}{\partial\phi}$.

5B. Evaluate the following limits

(i) $\lim_{x\to a}\frac{a^x-x^a}{x^x-a^x}$ (ii) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

5C. A plane passes through a fixed point (a, b, c). Show that the locus of the foot of the perpendicular from the origin on to the plane is the sphere,
$x^2+y^2+z^2-ax-by-cz=0.$

(3 + 4+ 3)

6A. Find the region of convergence of the following power series.

(i) $1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+....$

(ii) $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+...$

6B. Find the volume of the solid obtained by revolving the curve $y^2(2a-x)=x^3$ about its asymptote.

6C. If the sides of a plane triangle ABC vary in such a way that its circum – radius remains a constant, then prove that
$$\frac{\delta a}{\cos A}+\frac{\delta b}{\cos B}+\frac{\delta c}{\cos C}=0$$

(4 + 3+ 3)

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