(4242)

2009-2010

B.A./B.Sc. (Hons.) (PART – I) EXAMINATION

(MATHEMATICS)

VECTOR ANALYSIS AND GEOMETRY

(MM – 105)

Maximum Marks: 40 Duration: Three Hours

Note: Answer all questions.
All questions carry equal marks.

1. (a) Chords of the parabola $y^2 = 4ax$ subtend a right angle at the vertex. Find the locus of their middle points.

(b) Define diameters, conjugate diameters and find the condition that the lines $Ax^2 + 2Hxy + By^2 = 0$, may the conjugate diameters of the conic $ax^2 + 2hxy + by^2 = 1$,

OR

(b′) Pair of tangents are drawn to the conic $\alpha x^2 + \beta y^2 = 1$, so as to be always parallel to conjugate diameters of the conic $ax^2 + 2hxy + by^2 = 1$. Find the locus of their point of intersection.

2 (a) Trace the conic
$16x^2 - 24xy + 9y^2 + 77x - 64y + 95 = 0,$

OR

(a′) Find the foci and the eccentricity of the conic
$x^2 + 4xy + y^2 - 2x + 2y - 6 = 0,$

(b) Find the equation of the asymptotes of the conic $\frac{l}{r} = 1 + e\cos\theta$.

3 (a) Show that the equation of the right circular cylinder described on the circle through the points A: (1,0,0), B: (0,1,0) and C: (0,0,1) as the guiding curve is
$$x^2 + y^2 + z^2 - yz - zx - xy = 1$$

(b) Find the equation to the cone whose vertex is (α, β, γ) and the base the parabola $z^2 = 4ax$, $y = 0$

OR

(b′) Find the semi-vertical angle of a right circular cone which has three mutually perpendicular tangent planes.

Contd...2

4 (a) A section of surface is obtained by cutting it through a plane. Find the locus of the centres of the section of the surface $ax^2 + by^2 + cz^2 = 1$ if the plane touches $\alpha x^2 + \beta y^2 + \gamma z^2 = 1$

OR

(a′) Prove that the pole of the plane through the extremities of three conjugate semi-diameters of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ lies on the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 3$

(b) Find the locus of the point of intersection of three mutually perpendicular tangent planes to the surface $ax^2 + by^2 = 2z$

5 (a) If $\bar{a}, \bar{b}, \bar{c}$ are non zero vectors and $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$, then show that $(\bar{a} \times \bar{c}) \times \bar{b} = 0$. Moreover for four vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ show that $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = ((\bar{c} \times \bar{d}) \cdot \bar{a})\bar{b} - ((\bar{c} \times \bar{d}) \cdot \bar{b})\bar{a}$

(b) Define scaler field, Vector field and gradient of a scaler field and show that $\frac{df}{ds} = \overline{\nabla} \cdot \frac{d\bar{r}}{ds}$ and deduce from the above relation that the vector $\overline{\nabla}$ points in the direction in which $\frac{df}{ds}$ has maximum value, also this maximum value is equal to $|\overline{\nabla} f|$

OR

(b′) Define the divergence and curl of a vector field and show that $\overline{\nabla} \times (\overline{\nabla} f) = 0$ and $\overline{\nabla} \cdot (\overline{\nabla} \times \bar{a}) = 0$

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