(4243)

2009-2010

B.A./B.Sc. (Hons.) (PART – I) EXAMINATION

(MATHEMATICS)

NUMERICAL ANALYSIS

(MM – 106)

Maximum Marks: 40 Duration: Three Hours

Note: Answer all questions.

1. (a) Decompose the matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ into the LU form with $l_{ii} = 1, i = 1,2,3$ and solve the system AX=b where $b = [4 \quad 8 \quad 10]^T$. Determine L^{-1} and U^{-1} and hence find A^{-1} (4,4)

(b) Using Newton-Raphson method obtain a root correct to three decimal places of the equation x-cosx = 0

OR

(b') Determine the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ with initial vector $A = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

2 (a) Values of x (in degrees) and Sin x are given in the following table (4,4)

x	Sin x
15	0.2588190
20	0.3420201
25	0.4226183
30	0.5
35	0.5735764

Find the values of

(i) Sin 16 and Sin 32 using Newton Gregory formulae

(ii) Sing 26 using first three terms of Stirling's formula

OR

(a') Using the following data

x	y = f(x)
-1	3
0	-6
3	39
6	822
7	1611

Find x at y = 45 and 825 by inverse interpolation formula.

Contd.....2

(b) Show that

(i) $E^{-1} \equiv 1 - \nabla$

(ii) $\Delta^r y_k = \nabla^r y_{k+r} = \delta^r y_{k+r/2}$

(iii) $\Delta \nabla y_k = \nabla \Delta y_k = \delta^2 y_k$

3 (a) Evaluate $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$ (4,4)

(i) Using $\frac{3}{8}$ – Simpson's rule three times

(ii) Using Weddle's rule once

OR

(a′) The following table of values of x and y is given by

x	y
1	7.4036
2	7.7815
3	8.1291
4	8.4510

Find (i) $\frac{dy}{dx}$ when x = 1, x = 4.5

(ii) From this table, find x, correct to two decimal places for which y is maximum

(b) Derive Simpson's $\frac{3}{8}$ - rule

$$\int_{x_0}^{x_3} y\,dx = \frac{3}{8} h \left(y_0 + 3y_1 + 3y_2 + y_3\right)$$

Using this rule, evaluate

$$I = \int_0^1 \frac{1}{1+x} dx \text{ with } h = \frac{1}{6}.$$

4 (a) Given $\frac{dy}{dx} = 1 + y^2$, where y = 0 when x = 0, the Runge-Kutta method of fourth order produces the following values of y at x = 0.2, 0.4 and 0.6 (4,4)

x	y
0.2	0.2027
0.4	0.4228
0.6	0.6841

Use this data and compute the value of y at x = 0.8 using Adams-Moulton method.

OR

Contd....3