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**MANIPAL INSTITUTE OF TECHNOLOGY  
MANIPAL UNIVERSITY, MANIPAL - 576 104**



**FIRST SEMESTER B.E DEGREE MAKE UP EXAMINATION- DECEMBER 2010**

**SUB: ENGG. MATHEMATICS I (MAT – 101)  
(REVISED CREDIT SYSTEM)**

**Time : 3 Hrs.**

**Max.Marks : 50**

- Note :** a) Answer any FIVE full questions.  
b) All questions carry equal marks

1A. Find the  $n^{\text{th}}$  derivatives of the following

(i)  $\frac{x^2}{2x^2 + 7x + 6}$                       (ii)  $\cosh x \cdot \cos 3x$

1B. Trace the following curve with explanations  $y^2 (a-x) = x^3, a > 0$

1C. Find the image of the line  $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{4}$  in the plane  $2x + y + z = 6$ .  
(4 + 3+3)

2A. If  $y = \sin^{-1} x$ , show that  $(1-x^2)y_{n+2} = (2n+1)x y_{n+1} + (n^2 - m^2) y_n$

2B. Obtain the reduction formula for  $\int \sin^n x \, dx$  and hence evaluate  $\int_0^{\pi/2} \cos^n x \, dx$ .

2C. A variable plane at a constant distance  $p$  from the origin meets the coordinate axes at A, B, C. Through A, B, C planes are drawn parallel to coordinate planes. Show that locus of their point of intersection is  $x^{-2} + y^{-2} + z^{-2} = p^{-2}$ .  
(4 + 3+ 3)

3A. Find the nature of the series

(i)  $\sum_{n=1}^{\infty} \frac{n!2^n}{n^n}$                       (ii)  $\frac{x}{1} + \frac{1}{2} \frac{x^3}{3} + \frac{1.3}{2.4} \frac{x^5}{5} + \frac{1.3.5}{2.4.6} \frac{x^7}{7} + \dots$

3B. Sketch and find perimeter of the curve  $r = a (1 - \cos\theta), a > 0$

3C. Find the evolute of  $y^2 = 4ax$ .  
(4 + 3+ 3)

4A. Evaluate :

(i)  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x}{2} \right)^{1/x}$

(ii)  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2 \tan x}$

4B. Find the angle between the curves

$$r^m = a^m \cos m\theta, \quad r^m = a^m \sin m\theta, \quad a > 0 .$$

4C. Find the centre and the radius of the circle of intersection by the plane  $x+4y+z = 4$  and the sphere  $x^2 + y^2 + z^2 - x - z - 2 = 0$  .

(4 + 3+ 3)

5A. Find the first three nonzero terms in the Maclaurin's series expansion  $\tan x$ .

5B. The tangents at two points P, Q on the curve  $x = a (\theta - \sin\theta)$ ,  $y = a (1 - \cos\theta)$  are at right angles. Show that if  $\rho_1$  and  $\rho_2$  be the radii of curvature at the points, then show that  $\rho_1^2 + \rho_2^2 = 16a^2$  .

5C. Find the volume of the solid generated by revolution of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$  about the x - axis.

(4 + 3+ 3)

6A. (i) If  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$  then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u$$

6B. State and prove Lagrange's mean value theorem.

6C. Find the maximum possible error in calculating g if  $T=2\pi\sqrt{\frac{l}{g}}$  , given 1% and 0.5% errors in l and T respectively.

(4 + 3+ 3)

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