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GUJARAT UNIVERSITY

B. E. Sem III (C.E. (New) / I.T.) (New) Examination

Advance Mathematics-II

Saturday, 5th January, 2008] **[Time : 3 Hours**

Max. Marks : 100

Instructions : (1) Attempt all questions.
(2) Answer to the two sections must be written in separate answer books.
(3) Assume suitable data if required.
(4) Figures to the right indicate full marks.

SECTION I

1 Attempt any **three** : 18

(a) Express $f(x) = \frac{1}{2}(\pi - x)$ in a fourier series in the interval $0<x<2\pi$.

(b) Find the fourier series expansion for periodic function f(x), if $f(x) = -\pi \; ; \; -\pi<x<0$
$= x \; ; \; 0<x<\pi$

state the value of the series at x = 0 and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

(c) Obtain the fourier series to represent the function $f(x) = \pi^2 - x^2, \quad -\pi \le x \le \pi$.

Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$.

(d) If $f(x) = x^2; 0< x <2$ then find half range cosine series.

2 (a) Solve the following differential equations (any **two**) : 8

(i) $(D^2 - 4D + 4)y = e^{2x} + x^3 + \cos 2x$

(ii) $(D^2 + 5D + 6)y = e^{-2x} \sin 2x$

(iii) $(D^2 - 4D + 3)y = \sin 3x \cos 2x$

(b) By using the method of variation of parameters solve : $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ 4

OR

(b) Solve the simultaneous equations : 4

$\frac{dx}{dt} + 2x - 3y = t, \quad \frac{dy}{dt} - 3x + 2y = e^{2t}$

(c) Solve : $x^2 \frac{d^2y}{dt^2} - x\frac{dy}{dx} + 3y = x^2 \log x$ 4

OR

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(c) The charge 'Q' on the plate of a condenser of capacity 'C' charged through a resistance 'R' by a steady voltage 'V' satisfies the differential equation. **4**

$R\frac{dQ}{dt} + \frac{Q}{C} = V$ If $Q = 0$ at $t = 0$, show that $Q = CV[1-e^{-t/RC}]$. Find the current flowing into the plate.

3 (a) Solve in series the differential equation : **6**

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

OR

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2y = 0$$

(b) Attempt any **two** parts : **10**

(i) State cayley - Hamilton theorem and verify cayley-Hamilton theorem for the matrix.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

(ii) Using cayley -Hamilton relation obtain the inverse of the matrix.

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(iii) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ into canonical form.

SECTION II

4 (a) Form the partial differential equation from the following (any **one**) : **4**

(i) $z = (x^2 + a)(y^2 + b)$ (ii) $F(x^2 + y^2 + z^2, xyz) = 0$

(b) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\pi/2$ **4**

(c) Attempt any **two** : **8**

(i) $x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2)$

(ii) $p - q = x^2 + y^2$

(iii) $z^2(p^2x^2 + q^2) = 1$

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5 (a) Using the method of separation of variables solve : **4**

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \text{ and } u = e^{-5y} \text{ when } x = 0.$$

(b) Attempt any **three** : **12**

(i) Prove that the function sinhz is analytic and find its derivative.

(ii) Find the image of $|z - 3i| = 3$ under the mapping $\omega = {}^1/_z$.

(iii) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic.

(iv) Construct the analytic function f(z) of which the real part is $e^x \cos y$.

6 Attempt any **three** parts : **18**

(i) Find bilinear transformation which maps the points z = 1, i, -1 on to the points ω = 0,1, ∞.

(ii) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the paths (a) y = x (b) $y = x^2$.

(iii) Use cauchy's where integral formula to evaluate $\oint_C \frac{e^{-2z}}{(z+1)^3} dz$, where C is the circle $|z| = 2$.

(iv) Evaluate $\oint_C \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices :

(a) $2 \pm i$, $-2 \pm i$ (b) $-i$, $2 - i$, $2 + i$, i.

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Candidate's Seat No. ....

GUJARAT UNIVERSITY
B.E. Semester III (CE (Old) / IT (Old))
Advance Mathematics II

Saturday 5th January, 2008] **[Time: 3 Hours**
[Total Marks: 100

Instructions : (1) All questions are compulsory.
(2) Figure to the right indicate full marks.
(3) Attempt all questions from each section.
(4) Answer to the two sections must be written in separate answer sheet

SECTION I

1 Attempt **any three** 18

(a) Find the Fourier Series of the function, $F(x) = x + x^2; -\pi < x < \pi$
Hence deduce that,

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$$

(b) Find Fourier Series for $F(x) = x, \quad 0 \le x \le \pi$
$= 2\pi - x, \quad \pi \le x \le 2\pi$.

(c) Find the Fourier series for $F(t) = 1 - t^2, \ -1 \le t \le 1$.

(d) Expand $F(x) = x$ as a half range sine series in $0 < x < 2$.

(e) Obtain the Fourier series of sinh ax in $-\pi < x < \pi$.

2 (a) Solve **any two** of the following Differential Equations: 6
(i) $(D^2 - 6D + 13)y = 8e^{3x}\sin 4x + 2^x$
(ii) $(D^2 + 3D + 2)\,y = x\sin 2x$
(iii) $(D^2 + 6D + 9)y = e^{-3x}/x^3 + \cos 2x$

(b) Apply the method of variation of parameters to solve Differential Equation 5
$(D^2 + 4)y = 4\sec^2 2x$

OR

(b) Solve the following Simultaneous Differential Equation 5

$$\frac{d^2x}{dt^2} + 4x + 5y = t^2 \qquad \frac{d^2y}{dt^2} + 5x + 4y = t + 1$$

(c) Solve the Differential Equation 5

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$$

OR

(c) The Differential Equation of a circuit is $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$ 5
Solve the equation with initial conditions that $q = q_0$ and $dq/dt = 0$ when $t = 0$ and $CR^2 < 4L$.

3 (a) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, and hence find A^{-1} 6

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(b) Attempt any two: 10

(i) Define a unitary matrix. If $A=\begin{bmatrix}0 & 1+2i\\ -1+2i & 0\end{bmatrix}$, then show that $(I-A)(I+A)^{-1}$ is a unitary matrix, where I is an identity matrix.

(ii) Reduce the Quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_1x_3$ to canonical form and find the corresponding linear transformation.

(iii) Diagonalize the matrix $\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$, by orthogonal transformation.

SECTION II

4 (a) Construct any one Partial Differential Equation 4

(i) $z=(x^2+a)(y^2+b)$ (ii) $z=xy+F(x^2+y^2)$

(b) Solve any two Partial Differential Equation 8

(i) $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

(ii) $(1-x)p+(2-y)q=3-z$

(iii) $py+qx=pq$

(c) Using the method of separation of variable, Solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$, given $u(x,0)=6e^{-3x}$ 4

5 Attempt any three: 18

(a) Define Analytic function, If F(z) is Analytic Function then prove that

(i) $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|F(z)|^2=4|F'(z)|^2$

(ii) If $F(z)=u(x,y)+i\,v(x,y)$ prove that u and v are orthogonal to each other.

(b) Show that $u(x,y)=e^{-2xy}\sin(x^2-y^2)$ is a harmonic function. Find the harmonic conjugate and an analytic function F(z).

(c) Derive C-R Equation in Polar form. If $u=r^2\cos 2\theta$, find Analytic function F(z).

(d) An electrostatic field in the XY-plane is given by the Potential Function $\Phi=3x^2y-y^3$. Find stream line function.

(e) Find the Analytic function $F(z)=u+iv$, if $u-v=e^x(\cos y-\sin y)$.

6 (a) Solve $\frac{\partial^2 z}{\partial x\partial y}=a^2 z$, given that when $x=0$, $\frac{\partial z}{\partial x}=a\sin y$ and $\frac{\partial z}{\partial y}=0$ 4

(b) Attempt any three 12

(i) Show that the transformation $w=z+1/z$ maps the circle $r=c$ of z-plane into a family of ellipse in w-plane discuss the case when $r=1$.

(ii) Determine the region of the w-plane into which the region $\frac{1}{2}\le x\le 1$ and $\frac{1}{2}\le y\le 1$ mapped by transformation $w=z^2$.

(iii) If $w=1/z$ (A) Find the image of square whose vertices are 1+i, 4+i, 4+4i, 1+4i.
(B) Find the image of line $y=2x$ and $x+y=6$.

(iv) Define bilinear transformation, find bilinear transformation which maps the point $z=0, 1, \infty$ into point $w=-5, -1, 3$, respectively. What are the invariant points of the transformation?

(v) Show that the transformation $w=(2z+3)/(z-4)$ maps the circle $x^2+y^2-4x=0$ onto the straight line $4u+3=0$.

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