Seat No. : _____

FBCA-04

April-2007

Advanced Mathematics

(New Course)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (i) There are **five** questions.
(ii) **All** questions carry **14** marks.
(iii) Draw the figures wherever required.
(iv) Use of simple calculator is permitted.

1. (A) Define following terms : (any **four**) **(4)**
   (1) Intersection set.
   (2) Subset.
   (3) Quadratic function.
   (4) Break–Even Point.
   (5) Many–one function.

   (B) If A and B are two sets, then prove that the number of element $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ with Venn diagram. **(4)**

   OR

   If A, B and C are three sets then, prove that $A - (B \cup C) = (A - B) \cap (A - C)$ by usual notations. **(4)**

   (C) Attempt the following : (any **two**) **(6)**

   (1) If $U = \{x/1 \leq x \leq 8,\ x \in N\}$ $A = \{x/x \leq 4, x \in N\}$,
   $B = \{x/1 < x < 7, x \text{ is even no.}\}$ and $C = \{1, 2, 5\}$ then find
   (i) $A \cup (B - C)$
   (ii) $A \Delta B$
   (iii) $B \times C$
   (iv) $(A \cap B)'$

   (2) If $f(x) = \left[\frac{1-x}{1+x}\right]$, $x \in R$, then prove that $f(x) + f(1/x) = 0$.

(3) The fixed cost of a factory is Rs. 90,000 and the variable cost per unit of production is Rs. 150. If the selling price per unit is Rs. 240, then find :

(1) Revenue and cost function.

(2) Break–Even Point.

(3) If selling price is increased by Rs. 10, then find new Break–Even Point.

2. (A) When f (x) is said to be continuous at $x = a$? Also check the continuity of f(x) at $x = 5$. **(4)**

$$f(x) = \frac{x^2 - 25}{x - 5}, \quad x \leq 5$$
$$= 5, \quad x = 5$$
$$= 2x - 5, \quad x > 5$$

(B) Define following terms : (any **four**) **(4)**

(1) Matrix.

(2) Square Matrix.

(3) Row–Column Matrix.

(4) Transpose of Matrix

(5) Identity Matrix.

(C) Solve following problems (any **two**) : **(6)**

(1) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$ then

(i) Compute AB.

(ii) Is BA defined ?

(2) If $P = \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find Matrix 'X' if $3P + 5Q + 2X = 0$.

(3) If $A = \begin{bmatrix} 4 & 1 & 3 \\ 2 & 0 & 5 \\ 1 & 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 4 & 3 \\ 2 & 1 & 5 \end{bmatrix}$, then prove that

(i) $(A + B)^T = A^T + B^T$

(ii) $A + A^T$ is a symmetric matrix.

3. (A) Evaluate following limits : (any **two**) **(4)**

(i) $\lim_{x \to 4} \frac{x^3 - 64}{2x^2 - 32}$

(ii) $\lim_{x \to 0} \frac{7^{2x} - 5^{3x}}{x}$

(iii) $\lim_{x \to 3} \frac{\sqrt{x} - \sqrt{3}}{\sqrt{x+1} - 2}$

(iv) $\lim_{n \to \infty} \left[1 - \frac{2n}{5}\right]^{3/n}$

(B) Find the equation of a line parallel to $x - 2y + 3 = 0$ and passing from $(2, -3)$. **(4)**

OR

Find the equation of the line passing through the points A(3, –7) and B (–4, 9).

(C) Attempt the following : (any **two**) **(6)**

(1) Prove that the points (7, 0), (6, –2), (3, 4) and (4, 6) formed a parallelogram.

(2) Find the area of ΔABC whose vertices are A(2, 3), B(8, 5) and C (4, 7).

(3) Find angle between the line $5x - y + 2 = 0$ and $2x - 3y + 3 = 0$.

4. (A) Find the area bounded by x–axis and the curve $y = x^2 - 3x + 2$. **(4)**

(B) A company has the total cost $C = 500 + \frac{1}{2} X^2$ and the total revenue $R = 200x$ for x unit of production. So find **(4)**

(i) Total units for maximum profit.

(ii) Total maximum profit.

(C) Find dy/dx with respect to x (any **three)** **(6)**

(1) $y = 2^x + \log 2 + \frac{1}{x^2}$

(2) $y = \sqrt{4x^2 - 5}$

(3) $y = e^x . \tan x$

(4) $y = \frac{x^3}{\log x}$

(5) $x^2 + y^2 = 2xy$

5. (A) (i) Define Order and Degree of differential equation.

(ii) Give Order and Degree of following Diff. equation. **(4)**

(1) $\left(\frac{d^3y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + 2y = 0.$

(2) $\sqrt{\frac{d^2y}{dx^2}} = 3\frac{dy}{dx} + x$

(3) $x^2\frac{d^2y}{dx^2} + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$

(B) Attempt the following (any **two**) : **(4)**

(i) Solve $\frac{dy}{dx} = \frac{3+x}{3+y}$

(ii) Solve $(2x + 3y + 5)\,dx + (3x + 5y + 7)\,dy = 0$

(iii) Show that $y = Ax^2 + Bx$ is a solution of $\frac{d^2y}{dx^2} - \frac{2}{x}\cdot\frac{dy}{dx} + \frac{2y}{x^2} = 0$

(C) Evaluate following integrals (Any **three**) : **(6)**

(1) $\int \sqrt[3]{x} + 5 + 2/x \; dx$

(2) $\int \frac{3x^2}{\sqrt{x^3 - 1}}\, dx$

(3) $\int \frac{2x + 5}{(x + 2)(x + 3)}\, dx$

(4) $\int_1^2 (3x - 2)^2\, dx$

(5) $\int_0^{\pi/2} \cos^8 x \; dx$

Seat No. : _____

FBCA-04

April-2007

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(Old Course)

Time : 3 Hours] **[Max. Marks : 50**

Instructions : (1) Figures to the right indicate full marks.

(2) Scientific Calculator is not allowed.

1. (a) If A, B and C be any three sets, then prove that $A - (B \cap C) = (A - B) \cup (A - C)$. **(4)**

(b) Attempt any **two** parts : **(6)**

(1) (i) If A = {1, 2, 3, 4} and B = {4, 5,}, find $A \Delta B$ and $A \times B$.

(ii) If A = {a, b, c}, B = {b, d}, C = {b, c}, then verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(2) If the daily cost of production for x units of a manufactured product is given by $c(x) = 15x + 15{,}000$. Answer the following :

(i) If each unit is sold for Rs. 25, determine the minimum number of units that should be produced and sold to ensure no loss.

(ii) If the selling price is decreased by Rs. 5, per unit what would be the break–even point ?

(3) If $f(x) = x^5 - 2x + \frac{1}{x}$, prove that $f(x) + f(-x) = 0$.

2. (a) Find maximum and minimum value of the function $f(x) = 2x^3 + 9x^2 - 60x + 25$. **(4)**

(b) Attempt any **two** parts : **(6)**

(1) Evaluate :

(i) $\lim\limits_{x \to 3} \dfrac{x^3 - 27}{x - 3}$

(ii) $\lim\limits_{n \to \infty} \dfrac{n^2 + 2n - 1}{(n + 1)(2n + 1)}$

(2) Show that the function

$$f(x) = \begin{cases} \dfrac{2}{5-x} & , \quad x < 3 \\ 5 - x & , \quad x \geq 3 \end{cases}$$ is

(i) discontinuous from the left at $x = 3$.

(ii) Continuous from the right at $x = 3$.

(3) Differentiate the following w.r.t. x.

(i) $y = \dfrac{e^{2x}}{x^2 + 2x + 1}$

(ii) $y = e^x \left[(4x - 1)^2\right]$

3. (a) Write the reduction formula of $\int_0^{\pi/2} \text{Sin}^n x \, dx$. Hence evaluate $\int_0^{\pi/2} \sin^8 x \, dx$. **(4)**

(b) Evaluate the following integrals (any **three**) : **(6)**

(i) $\int \dfrac{x^{7/2} + x^8 + 1}{x^{5/2}} \, dx$

(ii) $\int x \cdot \log x \, dx$

(iii) $\int \dfrac{1}{(x+1)(x-2)} \, dx$

(iv) $\int_{-2}^{-1} \left(\dfrac{1}{x^2} - \dfrac{1}{x^3}\right) dx$

4. (a) Find the equation of a straight line which makes intercepts of a and b on x–axis and y–axis respectively. **(4)**

(b) Attempt any **two** parts : **(6)**

(i) Find the equation of lines passing through the intersection of $4x - 3y - 1 = 0$ and $2x - 5y + 3 = 0$ and perpendicular to $5x + 4y = 6$.

(ii) In what ratio is the line joining the points A(4, 4) and B(7, 7) divided by P(–1, –1) ?

(iii) Show that the points (4, –5), (8, 1), (14, – 3) and (10, –9) are the vertices of a square.

5. (a) Obtain the order and degree of the following differential equations (any **two**) : **(4)**

(i) $(2x + 3)\ \frac{d^3y}{dx^2} + \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2$

(ii) $\sqrt{\frac{d^2y}{dx^2}} = 5\frac{dy}{dx}$

(iii) $\left(\frac{d^4y}{dx^3}\right)^5 + \left(\frac{d^2y}{dx^2}\right)^3 = 3y$

(b) Solve the following differential equations (any **two**) : **(6)**

(i) $\frac{dy}{dx} + 5y = e^{-x}$

(ii) $(2x + 3y + 5)\,dx + (3x + 5y + 6)\,dy = 0$

(iii) $(x^2 + y^2)\frac{dy}{dx} = xy$