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Reg. No. : ....

Name : ....

Final Year B.Sc. Degree Examination, March 2009
Part – III : Group – I : MATHEMATICS
Paper IV – Differential Equations, Numerical Analysis and Vectors
(Perior to 2006 Admission)

Time : 3 Hours Max. Marks : 65

Instruction** : Maximum of **13** marks can be **earned** from **each Unit.

UNIT – I

1. Solve $\frac{dy}{dx} = \frac{y - x + 1}{y - x + 5}$. **4**

2. Show that the equation $(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$ is exact and hence solve it. **4**

3. Find the orthogonal trajectories of the circles $x^2 + (y - c)^2 = c^2$. **4**

4. Solve : $(D^2 - 1)\,y = 2x^2$. **4**

5. Solve : $(D^2 - 2D + 2)\,y = e^x \cos 2x$. **4**

UNIT – II

6. Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x \log x$. **5**

7. Solve the system $\frac{dx}{dt} = x + y$

$$\frac{dy}{dt} = 4x + y.$$ **5**

P.T.O.

8. Find the Laplace transforms of

i) e^{-t} Cos 2t   ii) $4e^{5t} + 6t^3 - 3$ Sin 4t. **4**

9. Solve the equation $y''(t) + y(t) = t$, $y(0) = 1$, $y'(0) = -2$, using Laplace transforms. **6**

UNIT – III

10. Prove that $\Delta^n \text{Sin}(ax+h) = \left(2\text{Sin}\frac{ah}{2}\right)^n \text{Sin}\left[ax+h+\frac{n}{2}(ah+\pi)\right]$. **6**

11. Prove that i) $1+\Delta = E$

ii) $1-\nabla = E^{-1}$. **4**

12. The following data gives the melting point of an alloy of lead and zinc, where t is the temperature and p is the percentage of lead in the alloy.

p	:	60	70	80	90
t	:	226	250	276	304

Applying Newton's interpolation formula, find the melting point of the alloy containing 84 percent of lead. **5**

13. Apply Lagrange's formula to find f (5) given that f (1) = 2, f(2) = 4, f (3) = 8, f (4) = 16, f(7) = 128. **5**

UNIT – IV

14. Prove that $(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d}) = (\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{d}) - (\bar{a}\cdot\bar{d})(\bar{b}\cdot\bar{c})$. **4**

15. If $r = |\bar{r}|$, where $\bar{r} = xi + yj + zk$, prove that $\nabla\,\bar{r} = \frac{\bar{r}}{r}$. **4**

16. Find the directional derivative of the function $2xy + z^2$ in the direction of the vector $\bar{i} + 2\bar{j} + 2\bar{k}$ at the point (1, –1, 3). **4**

17. Show that $\overline{F} = (2xy + z^3)\overline{i} + x^2\,\overline{j} + 3xz^2\,\overline{k}$ is a conservative force field . Find the scalar potential. **4**

18. If $\overline{F}$ is any vector point function, prove that div (Curl $\overline{F}$) = 0. **4**

UNIT – V

19. Evaluate $\int_c \overline{F}\cdot dr$, where $\overline{F} = x^2 i + y^2 j$ and c is the arc of the parabola $y = x^2$ in the xy - plane from (0, 0) to (1, 1). **5**

20. Evaluate $\iint_s \overline{F}\cdot n\, ds$, where $\overline{F} = 6z\overline{i} - 4\overline{j} + y\overline{k}$, where s is the portion of the plane $2x + 3y + 6z = 12$ in the first octant. **6**

21. State Green's Theorem. **3**

22. Apply Stoke's theorem to evaluate $\int_C (y\,dx + z\,dy + x\,dz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + y = a$. **6**