

Reg. No.:....

Name:.....

Final Year B.Sc. Degree Examination, March 2009 Part – III: Group – I: MATHEMATICS Paper IV – Differential Equations, Numerical Analysis and Vectors (Perior to 2006 Admission)

Time: 3 Hours Max. Marks: 65

Instruction: Maximum of 13 marks can be earned from each Unit.

UNIT – I

1. Solve
$$\frac{dy}{dx} = \frac{y - x + 1}{y - x + 5}$$
.

- 2. Show that the equation $(x^2 4xy 2y^2) dx + (y^2 4xy 2x^2) dy = 0$ is exact and hence solve it.
- 3. Find the orthogonal trajectories of the circles $x^2 + (y c)^2 = c^2$.
- 4. Solve : $(D^2 1) y = 2x^2$.
- 5. Solve: $(D^2 2D + 2) y = e^x \cos 2 x$.

UNIT - II

6. Solve:
$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$$
.

7. Solve the system $\frac{dx}{dt} = x + y$

$$\frac{\mathrm{dy}}{\mathrm{dt}} = 4x + y.$$

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- 8. Find the Laplace transforms of
 - i) e^{-t} Cos 2t
- ii) $4e^{5t} + 6t^3 3 \sin 4t$.

9. Solve the equation y''(t) + y(t) = t, y(0) = 1, y'(0) = -2, using Laplace transforms.

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- 10. Prove that $\Delta^n \sin(ax+h) = \left(2\sin\frac{ah}{2}\right)^n \sin\left[ax+h+\frac{n}{2}(ah+\pi)\right]$.
- 11. Prove that i) $1 + \Delta = E$

60

ii) $1 - \nabla = E^{-1}$.

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12. The following data gives the melting point of an alloy of lead and zinc, where t is the temperature and p is the percentage of lead in the alloy.

p :

70

80

90

t: 226

250

276 30

Applying Newton's interpolation formula, find the melting point of the alloy containing 84 percent of lead.

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- 13. Apply Lagrange's formula to find f(5) given that f(1) = 2, f(2) = 4, f(3) = 8,
 - f(4) = 16, f(7) = 128.

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UNIT - IV

14. Prove that $(\overline{a} \times \overline{b}) \cdot (\overline{c} \times \overline{d}) = (\overline{a} \cdot \overline{c})(\overline{b} \cdot \overline{d}) - (\overline{a} \cdot \overline{d})(\overline{b} \cdot \overline{c})$.

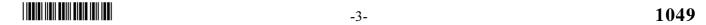
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15. If $r = |\bar{r}|$, where $\bar{r} = xi + yj + zk$, prove that $\nabla \bar{r} = \frac{\bar{r}}{r}$.

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16. Find the directional derivative of the function $2xy + z^2$ in the direction of the vector $\overline{i} + 2\overline{j} + 2\overline{k}$ at the point (1, -1, 3).

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17. Show that $\overline{F} = (2xy + z^3)\overline{i} + x^2\overline{j} + 3xz^2\overline{k}$ is a conservative force field. Find the scalar potential.

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18. If \overline{F} is any vector point function, prove that div (Curl \overline{F}) = 0.

- 19. Evaluate $\int_c \overline{F} \cdot dr$, where $\overline{F} = x^2 \mathbf{i} + y^2 \mathbf{j}$ and c is the arc of the parabola $y = x^2$ in the xy plane from (0, 0) to (1, 1).
- 20. Evaluate $\iint_s \overline{F} \cdot n \, ds$, where $\overline{F} = 6z\overline{i} 4\overline{j} + y\overline{k}$, where s is the portion of the plane 2x + 3y + 6z = 12 in the first octant.
- 21. State Green's Theorem.
- 22. Apply Stoke's theorem to evaluate $\int_C (y dx + z dy + x dz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and x + y = a.