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2010 – 2011
B.Sc. (HONS.) (PART – III) EXAMINATION
(PHYSICS)
MATHEMATICAL METHODS
(PH – 311)

Maximum Marks : 40 Duration : Three Hours

Note: Answer all questions.

1.(a) Find the analytic function f(z) if $u(x, y) = x^3 - 3xy^2$. [2½]

(b) Evaluate $\oint_C \frac{z\,dz}{z^2+4z+3}$, the curve C is given by | z | = 2. [2½]

(c) State and prove Cauchy's integral theorem. [2]

OR

1′.(a) Show that $\oint_C \frac{f(z)}{z-z_0}\,dz = 2\pi i\, f(z_0)$, where z_0 is a point in the interior region bounded by C and f(z) is analytic on C and within the region surrounded by C. [2]

(b) Find the residues of the function $\frac{z}{z^2+4}$ at each pole. [2]

(c) Evaluate the real integral [3]

$$\int_0^{2\pi} \frac{d\theta}{1+\epsilon\cos\theta}, \ |\epsilon|<1$$

using calculus of residues.

2.(a) Show that the gradient of a scalar function ϕ (x, y, z) is identified as a vector having the direction of the maximum space rate of change of ϕ. [4]

(b) State and prove Gauss's theorem. [2]

OR

2′.(a) Define curvilinear coordinates (q_1, q_2, q_3) system. Show that square of the distance ds between two neighbouring points in the orthogonal curvilinear coordinates can be written as $ds^2 = \sum_i (h_i\ dq_i)^2$, where h_i's are scale factors. [4]

(b) Show that [2]

$$\vec{\nabla}\cdot[\vec{r}\ V(r)] = 3V(r) + r\frac{dV(r)}{dr}$$

where V(r) is a central potential.

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3.(a) Evaluate $\int_{-\infty}^{+\infty} e^{-x^2} H_n^2(x)\,dx$, where $H_n(x)$ is Hermite polynomials. [4]

(b) Show that Bessel's functions $J_n(x)$ and $J_{-n}(x)$ are linearly dependent for integer n. [3]

4.(a) Show that $y_{l,-m}(\theta,\phi) = (-1)^m\, y^*_{l,+m}(\theta,\phi)$. [3]

(b) For associated Legendre polynomials $P_p^m(x)$, show that [4]

$$\int_{-1}^{+1} P_p^m(x) P_q^m(x)\,dx = \frac{2}{2q+1}\frac{(q+m)!}{(q-m)!}\,\delta_{pq}.$$

5. Solve the wave equation for the vibration in a circular membrane of radius R, clamped at the circumference, if its initial deflation u (r, 0) = f(r) and velocity $\left.\frac{\partial u}{\partial t}(r,t)\right|_{t=0} = g(r)$. Discuss the normal modes and sketch the modes for $m=1$ and $m=2$ [7]

6.(a) Solve the differential equation [2]

$$\frac{d^2y(t)}{dt^2} - y(t) = t$$

using Laplace transform, if y(0) = 1 and y′(0) = 1.

(b) Solve the integral equation [4]

$$y(x) = 1 + \lambda \int_0^1 (1-3xz)\, y(z)\, dz.$$

OR

6′.(a) Classify the integral equations and give the example for each case. [3½]

(b) Find the Fourier transform of e^{-ax^2} (where a > 0). [2½]
