4292

2008-2009
B.Sc.(HONS.) (PART-III) EXAMINATION
(PHYSICS)
MATHEMATICAL METHODS
(PH-307)

Maximum Marks: 40 Duration : Three Hours.

Answer all questions.
Marks are indicated against each part.

1.(a) Define complex integral. If f (z) is an analytic function in a simply connected domain D, then prove that

$$\oint_C \frac{f(z)}{z-z_0}\,dz = 2\pi i f(z_0)$$

where z_0 is some point in the interior region bounded by curve C in domain D. 04

(b) Develop the Taylor series expansion of $\ell n(1+z)$ for $|z|<1$. 03

2.(a) State and prove Divergence theorem of Gauss . 04

(b) Prove that line integral

$$\int_C \vec{F}(r)\cdot dr = \int_C (F_1\,dx + F_2\,dy + F_3\,dz)$$

is independent of path if and only of $\vec{F} = [F_1, F_2, F_3]$ is a gradient of some function f in D i.e. $\vec{F}$ = grad f. 03

OR

2'(a) Show that the scale factors h_j corresponding to curvilinear coordinates q_j for orthogonal system are given by the relation

$$h_j^2 = \sum_{k=1}^{3}\left(\frac{\partial x_k}{\partial q_j}\right)^2$$

where x_k are the Cartesian coordinates. 05

(b) Calculate the Spherical polar coordinates scale factors h_r, h_θ and h_ϕ. 02

3.(a) Define Gamma function in integral form and show that $\Gamma(\overline{n}+1) = n!$. 03

(b) Establish the following differential formula for Hermite polynomials: 03

$$H_n(x) = (-1)^n \frac{d^n(e^{-x^2})}{dx^n}.$$

....2

4.(a) For Legendre polynomial $P_n(x)$ prove that: 03

$$\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0 \quad \text{for} \quad n \neq m$$

(b) Show that $n P_n(x) = (2n-1) x P_{n-1}(x) - (n-1) P_{n-2}(x)$. 04

OR

4'(a) Find the Associated Legendre equation from Legendre differential equation. 03

(b) Find the value of Spherical harmonics Y_{10}. 04

5.(a) A triangular wave is represented by the function f (x) as

$$f(x) = \begin{cases} x & \text{if } 0 < x < \pi \\ -x & \text{if } -\pi < x < 0. \end{cases}$$

Represent f (x) by a Fourier series. 03

(b) Consider a thin circular plate whose faces are impervious to heat flow and whose circular edge is kept at zero temperature. At t = 0, the initial temperature of the plate is a function of the distance from the centre of the plate. Find the expression for the subsequent temperature. 04

OR

5'(a) Represent

$$f(x) = \begin{cases} 1 & 0 < x < \tfrac{1}{2} \\ 0 & \tfrac{1}{2} < x < 1. \end{cases}$$

in a Fourier series in series and cosines. 03

(b) A bar of length ℓ and uniform cross section whose surface is impervious to heat flow has an initial temperature F (x). Its ends are kept at the constant temperature zero. Determine the subsequent temperature of the bar as time t increases. 04

6.(a) Obtain the solution of the non-homogeneous Fredholm equation of second kind by separable Kernel method. 03

(b) Solve the equation

$$\phi(x) = 1 + \lambda \int_0^1 (1 - 3xt)\, \phi(t)\, dt$$