(4292)

2009–2010
B.Sc. (HONS.) (PART-III) EXAMINATION
(PHYSICS)
MATHEMATICAL METHODS
(PH – 307)

Maximum Marks : 40 — Duration : Three Hours

NOTE: (i) Answer ALL questions.
(ii) Marks are indicated against each part.

1. (a) State and prove Cauchy's integral theorem. (04)

(b) Assuming that f(z) is analytic on and within a closed contour c and that point z_o is within c, show that : (03)

$$\oint_c \frac{f'(z)dz}{z-z_0} = \oint_c \frac{f'(z)dz}{(z-z_0)^2}$$

1'. (a) Define an analytic function and obtain the Cauchy-Riemann differential equations. (04)

(b) Using Taylor expansion show that : (03)

$$\frac{1}{1-t} = \sum_{n=0}^{\infty} t^n$$

2. (a) State and prove Stoke's theorem of vector analysis. Show that Green's theorem in the plane is the special case of Stoke's theorem. (04)

(b) Show that gradient of a differentiable scalar function f at a point P of surface S. f(x,y,z)=constant, is a normal vector of S at P. (03)

3. (a) Define Beta and Gamma functions and show that : (03)

$$\Gamma(n)\,\Gamma(m) = \Gamma(m+n)\,\beta(m, n)$$

(b) Establish the following orthogonal property of Hermite polynomial : (03)

$$\int_{-\infty}^{\infty} e^{x^2} H_n(x) H_m(x) dx = \sqrt{\pi}\, 2^n n! \delta_{mn}$$

4. (a) Show that the coefficient of t^n in the expansion of the function $(1 - 2xt + t^2)^{-1/2}$ is the Legendre Polynomial $P_n(x)$ of degree n for $|x| < 1$. (03)

(b) Show that the function $\phi_n(x) = e^{\frac{-x^2}{2}} L_n(x); \quad n = 0, 1, 2, \ldots\ldots\ldots$ form an orthonormal set of functions in the interval $0 \le x \le \infty$ i.e. $\int_0^{\infty} \phi_n(x)\phi_m(x) = \delta_{mn}$. (03)

OR

Contd.....2

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4'. (a) Obtain a solution of Bessel's equation : (03

$$x^2y'' + xy' + (x^2 - n^2)\,y = 0$$

by series substitution method.

(b) Find the Rodrigue's formula for Legendre Polynomials. (03

5. (a) Find the Fourier coefficients of the periodic function : (04

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases} \quad \text{and } f(x+2\pi) = f(x)$$

Hence prove that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots\ldots\ldots\ldots$

(b) The vibration of an elastic string are governed by the one dimensional wave equation (03

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Where u(x, t) is the displacement of string. Find the displacement u(x, t) for boundary condition u(0, t) = u(L, t)=0 and u(x,0)=f(x) and $\frac{\partial u}{\partial t}\Big|_{t=0} = g(x)$.

OR

(b') Solve Laplace's equation $\nabla^2 v = 0$ where v represents the temperature at point (x, y) on a thin metal plate that is bounded by the lines x = 0, x = S, y = 0 and y = h subject to the boundary conditions v = 0 at x = 0 and x = S, v = 0 at y = 0, v = F(x) at y = h. (03

6. (a) A linear oscillator equation $y'' + \omega^2 y = 0$ with the boundary conditions y(0) = 0 and y(b)=0 is represented by an integral equation : (04

$$y(x) = \omega^2 \int_0^b k(x,t)\,y(t)\,dt$$

Find the kernel k (x, t).

OR

(a') Give the general procedure for solving Fredholm's equations of the second kind with separable kernels. (04

(b) Solve the equation : (03

$$\phi(x) = x + \tfrac{1}{2}\int_{-1}^{1} (t + x)\,\phi(t)\,dt$$
