4293

2008-2009
B.Sc.(HONS.) (PART-III) EXAMINATION
(PHYSICS)
CLASSICAL MECHANICS & SPL. RELATIVITY
(PH-308)

Maximum Marks: 40 Duration : Three Hours.

Answer the following questions.
Notations / symbols wherever not explained have their usual meanings.
Use of calculator is permitted.

1.(a) Derive the Lagrange's equations of motion for a conservation system using D' Alembert's principle. 05

(b) The position vectors of particles '1' and '2' of equal mass vary with time t as $\vec{r}_1 = \hat{i}\,(a + v\,t)$ and $\vec{r}_2 = \hat{j}\,(b + v\,t)$, where a and b are constants. Determine the velocity of their centre-of-mass. Find the relative velocity of particle '1' with respect to particle '2' and the magnitude of the relative velocity. 02'

OR

1'(a) State and prove the conservation theorem for the total angular momentum of a system of particles. 03

(b) What is a conservative force? Show that a conservative force $\vec{F}$ can be expressed as $\vec{F} = -\vec{\nabla} V$, where V is a scalar function of position coordinates. Write the potential energy function of a system of two charged particles and hence derive a formula for force between the two. 04

2. Derive the Euler – Lagrange's equation to find the function y (x) such that the line integral

$$J = \int_{x_1}^{x_2} f(y, y', x)\, dx \,, \quad \text{where } y' = \frac{dy}{dx}$$

has a stationary value.

Obtain the equation of the curve in a plane whose surface of revolution has a minimum area. 07

OR

2'(a) Define the Hamiltonian function H (q, p , t) and show that $\frac{dH}{dt} = -\frac{\partial L}{\partial t}$. Discuss the case when H represents the total energy of the system and is a constant of motion. 04

(b) Obtain the Hamiltonian of a particle whose Lagrangian is given by

$$L = \tfrac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) - V(r)$$

Determine its constants of motion. 03

3.(a) Show that the central force motion of two bodies about their centre –of – mass can be reduced to an equivalent one-body problem. 03

(b) Show that the conservation of angular momentum and the constancy of areal velocity of radial vector is a general result of any central force motion. 04

4.(a) Obtain the Hamilton's equations of motion. Using the-se equations, obtain the equation of motion of a linear harmonic oscillator. 03

(b) Discuss the elastic scattering of two equal mass particles with the target at rest in the lab system. Describe the scattering in the C.M.S. 03

5.(a) What is meant by tensors of the same type? Write transformation equations for the component of a tensor or contra-variant rank 2 and covariant rank 1 under the change of coordinates. Discuss the contraction of a tensor A^{ij}_{kl}. 03

Velocity defined as the rate of change of the position vector of a particle with respect to its proper time i.e. $u_\mu = \frac{dx_\mu}{d\tau}$.

(b) Define the four-velocity and four-force vectors of a particle and find the value of their scalar product. 02

(c) Calculate the kinetic energy (in MeV) and momentum (in MeV / c) of an electron moving with speed $\frac{c}{\sqrt{2}}$. 02

OR

5'(a) A particle 'a' of rest mass m_a and kinetic energy K_a collides with a particle 'b' of rest mass m_b and kinetic energy zero. Find a formula for energy in the C.M.S. 02

(b) Define threshold energy of a reaction and obtain its formula. Calculate the threshold kinetic energy of the incident particle for the following reaction:

$$p + p \rightarrow p + p + \pi^0.$$

the target is rest. Take $m_p = 938\ \text{MeV}/c^2$ and $m_{\pi^0} = 135\ \text{MeV}/c^2$. 03

(c) Derive a formula for the Lagrangian of a relativistic particle. 02

6.(a) Derive the transformation equations for the components of the electromagnetic field vectors $\vec{E}$ and $\vec{B}$ between two reference frames having uniform relative motion along the x- axis. 04

(b) Show that the equation of motion of a charged particle in an electromagnetic field remains invariant under the gauge transformation.

$$\vec{A} \rightarrow \vec{A} + \nabla\chi$$

and $$\phi \rightarrow \phi - \frac{1}{c}\frac{\partial \chi}{\partial t}$$

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