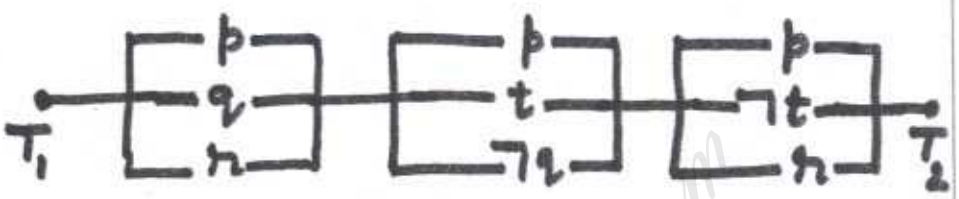
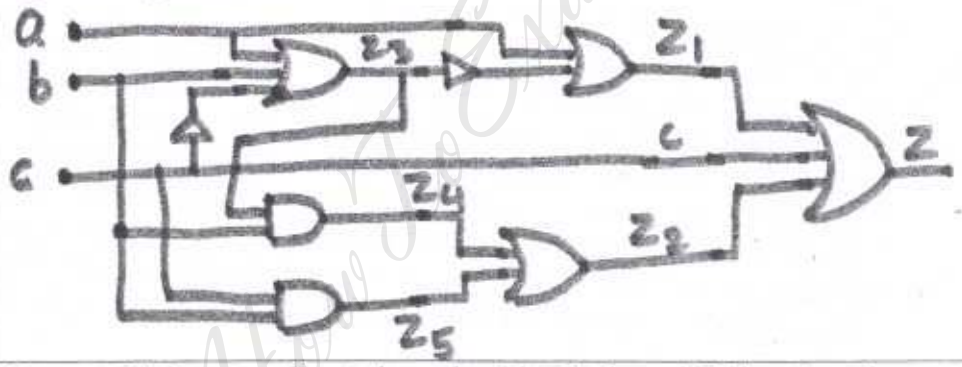
(9)

School of Mathematics and Computer Applications, T.I.E.T., Patiala
End semester Examination (December, 16, 2005)
Time: 3 Hours Discrete Mathematical Structure (CA-003) Max. Mark: 45

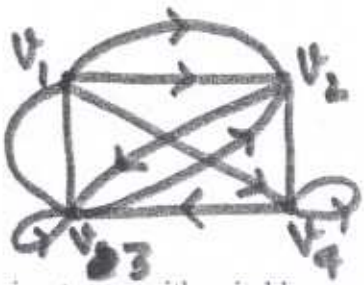
Note :(i) Answer any FIVE questions (ii) Answers of only first five questions will be checked (iii) Evaluated answer sheet shall be shown on December 10, 2006 at 3.00 P.M. in B-208.

Q. 1(a)	Simplify the following on-off network using standard logical equivalent formula and represent the simplified form in network. 	5
(b)	In calculus the definition of the limit L of a function $f(x)$ is given as $f(x) \to L$ as $x \to a$ if and only if for every $\varepsilon > 0$ there exists a $\delta > 0$, so that for all $x \in I\,(0 < \lvert x-a \rvert < \varepsilon)$ then $\lvert f(x)-L \rvert < \varepsilon$. Negate the expression $f(x) \to L$ as $x \to a$..	4
Q. 2(a)	Write the following argument in symbolic form using predicate and then verify the validity of the argument using predicate calculus by assuming all the adults (18 years or above) who are presently residing in Delhi as universe of discourse **"All credit union employees must know COBOl. All credit union employees who write loan applications must know Excel. Anita works for the credit union, but she does not know Excel. Mohan knows Excel but does not know COBOL. Therefore, Anita doesn't write loan applications and Mohan doesn't work for the credit union."**	5
(b)	Define reflexive and symmetric closures with suitable example. Let $A = \{1, 2, 3, 4\}$ and R be a relation such that $M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Obtain transitive closure of R using Warshall Algorithm.	4
Q. 3(a)	Let $S = \{x : x \in R \text{ and } x \neq 0,\ x \neq -1\}$. Consider $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ where $f_1(x) = x$, $f_2(x) = 1-x$, $f_3(x) = 1/x$, $f_4(x) = 1/(1-x)$, $f_5(x) = 1-1/(1-x)$, $f_6(x) = x/(x-1)$ are functions from S to S. Prove that (S, o) is a group where o is the composition of function.	5
(b)	Prove that intersection of any subgroups of a group is again a subgroup but their union in general. is not a subgroup.	4
Q. 4(a)	Let $(H, *)$ be a subgroup of a group $(G, *)$. Show that the relation	2.5

(b)	$R = \{(x, y) : x,\ y \in G,\ x^{-1} * y \in H\}$ is an equivalence relation on H.	3
(c)	Using generating function method, find the explicit formula for Fibonacci sequence.	
	Solve the following recurrence relation by the method of characteristic roots. $a_r - 6a_{r-1} + 8a_{r-2} = r \cdot 4^r$ where $a_0 = 8$, and $a_1 = 22$.	3.5
Q. 5(a)	For $X = \{0, 1\}$, let $A = X \times X$. Define relation R on A by $(a, b)R(c, d)$ if (i) $a < c$; or (ii) $a = c$ and $b \le d$. Prove that R is a partial order on A. Determine all minimal and maximal elements of this partial order.	3
(b)	Let $f : R \times R \to Z$ be the closed binary operation defined by $f(a, b) = \lceil a + b \rceil$. Is f associative? . Is f commutative? . Does f have an identity element? Justify your answer.	3
(c)	Let $(L, \le)$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. Prove that $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.	3
Q. 6(a)	Let a and b be elements of a Boolean Algebra. Prove that $(a \vee b)' = a' \wedge b'$.	4
(b)	Express the output Z as a Boolean expression of the logic circuit given below for which a, b, c, are inputs and simplify the Boolean expression algebraically to find DNF.	5



Q. 7(a)	Suppose G is a non-directed graph with 12 edges. If G has 6 vertices each of degree 3 and the rest have degree less than 3. What is the minimum number of degree of vertices G can have?	3
(b)	Write down the adjacency matrix of the following graph and find the in-degree and out-degree of each vertex using the adjacency matrix.	3



(c)	Explain the following terms with suitable example: (i) Multigraph (ii) Weighted graph (iii) Regular graph	3