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Thapar Institute of Engineering and Technology
School of Mathematics and Computer Applications
End Semester Examination- December 15, 2006
Mathematics-I (MA-101)

Time: 3 hours **Max. Marks. 45**

Note: Attempt any **FIVE** questions. Attempt all parts of each question in a sequence at one place. Do write your tutorial group on the top of your answer sheet.

Q1.(a) If $y=\sin(a\cos^{-1}\sqrt{x})$, then prove that

$$\lim_{x\to 0}\left(\frac{y_{n+1}}{y_n}\right)=\frac{4n^2-a^2}{4n+2}.$$

(b) For what values of a, m and b does the function

$$f(x)=\begin{cases} 3, & x=0 \\ -x^2+3x+a, & 0<x<1 \\ mx+b, & 1\le x\le 2 \end{cases}$$

satisfy the hypothesis of the Mean Value Theorem on the interval [0,2] ?

(c) Show that the point $\left(\frac{1}{2},\frac{3\pi}{2}\right)$ lies on the curve $r=-\sin(\theta/3)$ and hence find the slope of the curve at this point.

(3.5+3.5+2)

Q2. (a) State and prove the integral test.

(b) Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty}\left(\frac{1+\sin(a_n)}{2}\right)^n$ converges.

(c) For the function $z=f(x,y)$, transform the equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$ into polar coordinate system.

(3+2+4)

Q3. (a) Find the absolute maximum and minimum of the function

$f(x,y)=(4x-x^2)\cos y$ on the region $1\le x\le 3$ and $-\frac{\pi}{4}\le y\le\frac{\pi}{4}$.

(b) Obtain the relation $\tau=-\frac{d\vec{B}}{ds}\cdot\vec{N}$, for the torsion τ of a curve at any point, where symbols have their usual meanings.

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(c) At time $t=0$, a particle is located at the point (1,2,3). It travels in a straight line to the point (4,1,4), has speed 2 at (1,2,3) and constant acceleration $3\mathbf{i}-\mathbf{j}+\mathbf{k}$. Find an equation for the position vector $\mathbf{r}(t)$ of the particle at time t.

(2.5+2.5+4)

Q4. (a) Evaluate the following integral by making use of proper substitution

$$\iint_R (x-y)^4 e^{(x+y)}\,dx\,dy$$

where R is the square with vertices (1,0), (2,1),(1,2) and (0,1).

(b) Evaluate the double integral by changing the order of integration

$$\int_0^2 \int_{\sqrt{2x-x^2}}^{\sqrt{2x}} y\,dy\,dx.$$

(4+5)

Q5. (a) Evaluate the line integral $\int_C (\sin x - y)\,dx - \cos x\,dy$ where C is the triangle whose vertices are (0,0), $\left(\frac{\pi}{2},0\right)$ and $\left(\frac{\pi}{2},1\right)$.

(b) State tangential form of the Green's theorem in the plane and verify it for the integral $\int_C (x^2-2xy)\,dx + (x^2y+3)\,dy$ along the boundary of the region bounded by $y^2 = 8x$ and $x = 2$.

(4+5)

Q6. (a) Solve the differential equation $xy(1+xy^2)\frac{dy}{dx} = 1$.

(b) Find the solution of the differential equation $y\,dx - x\,dy + 3x^2y^2e^{x^3}\,dx = 0$.

(c) Use Taylor's formula to find the quadratic approximation of $e^x \sin y$ at the origin. Estimate the error in the approximation if $|x| \le 0.1$ and $|y| \le 0.1$.

(3.5+2.5+3)

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