

SCHOOL OF MATHEMATICS AND COMPUTER APPLICATIONS, T.I.E.T., Patiala
End Semester Examination First Semester 2006-07

Time: Three hours

Numerical Analysis (MA-202)

Max. Mark: 100

Note: (i) Answer any FIVE questions.(ii) All questions carry equal marks. (iii) Write your tutorial group on the top of the first page of your answer sheet

1(a)	What is the loss of significant figures? How do you overcome with this difficulty whenever it occurred during computation? Elaborate with appropriate example.	5
(b)	If in the formula $R = \frac{r^2}{2h} + \frac{h}{2}$, the percentage error in R is not to exceed 0.3%. Find the allowable percentage error in r and h , when $r = 48 \text{ mm}$ and $h = 56 \text{ mm}$.	(5)
2(a)	Prove that the order of convergence of Secant method is 1.618.	(5)
(b)	Show that $x = 1 + \tan^{-1} x$ has a zero in the interval $\left[1, 1 + \frac{\pi}{2}\right]$. Is this interval contains fixed - point such that for any $x_0 \in [a, b]$ the iteration function $x_{n+1} = 1 + \tan^{-1} x_n$, $n \geq 0$ will converge to unique solution? If yes, find the solution correct to three decimal places.	(5)
3(a)	If $f(x)$ be a function defined on $[a, b]$ and set of nodes follow $a = x_0 < x_1 < \dots < x_n = b$. Write all the six conditions so that $g(x)$ becomes the cubic spline interpolant for $f(x)$.	(4)
(b)	Given that x ; 1.0 1.5 2.0 $\log x$: 0.0 0.17609 0.30103 Find $\log 1.8$ using Newton divided interpolation formula	(6)
4(a)	Show that Gauss elimination method applied to a system of order n requires $n(n-1)/2$ divisions, $n(n^2-1)/2$ multiplications and $n(n^2-1)/2$ additions operations in reducing the system into triangular form.	(4)
(b)	Solve the following system of equations using Crout's triangularization method $x_1 + x_2 + x_3 = 1$; $2x_1 + 3x_2 - x_3 = 6$; $3x_1 + 5x_2 + 3x_3 = 6$	(6)

<p>5(a)</p> <p>(b)</p>	<p>State sufficient condition for the convergence of iterative method to solve system of linear equations. Working with four decimal digit rounding arithmetic, solve the following system of equations using Jacobi method correct to two decimal places by taking $X = [0 \ 0 \ 0]^T$</p> <p>$27x + 6y - z = 85; \ 6x + 15y + 2z = 72; \ x + y + 54z = 110$</p> <p>Using Power method, find the largest eigenvalue and corresponding eigenvector of the following matrix correct to two decimal places by taking $X_0 = [1 \ 0 \ 0]^T$</p> $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	<p>(5)</p> <p>(5)</p>
<p>6(a)</p> <p>(b)</p>	<p>Establish trapezoidal formula of numerical integration by integrating two-point Lagrangian interpolation formula.</p> <p>Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss two-point and three-point quadrature formulas and compare with the exact value of integral.</p>	<p>(5)</p> <p>(5)</p>
<p>7(a)</p> <p>(b)</p>	<p>Show that the local truncation error of improved Euler's method is $O(h^3)$.</p> <p>Given the values of $u(x, y)$ on the boundary of the square in the figure, evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of this figure by Gauss-Seidel method</p> <div style="text-align: center;"> <p>The diagram shows a square grid with four interior nodes labeled u_1, u_2, u_3, u_4. The boundary values are: top edge (1000, 1000, 1000, 1000), right edge (500, 0), bottom edge (0, 0, 500, 1000), and left edge (2000, 2000).</p> </div>	<p>(3)</p> <p>(7)</p>