THAPAR INSTITUTE OF ENGINEERING & TECHNOLOGY-PATIALA
END SEMESTER EXAMINATION (DEC. – 2006)

(04)

Optimization Techniques(MA-203) **M.M. 45 Time: 3 Hrs.**
Instructor-in-Charge : Rajesh Kumar

Note : Attempt any **five** questions. Do write your tutorial group on the top of answer sheet. Evaluated answer sheets will be shown on **18-12-2006** at 11:00AM (Room No. **B210**)

Q1.(a) A complete unit of a certain product consists of four units of component A and three units of component B. Two components (A and B) are manufactured from two different raw materials of which 100 and 200 units, respectively, are available. Three departments are engaged in the production process with each department using a different method for manufacturing the components. The following table gives the raw material requirements per production run and the resulting units of each component. The objective is to determine the numbers of production runs for each department which will maximize the total numbers of components units of the final product. Formulate the problem.

	Input/run (units)		Output/run (units)	
Department	Raw material 1	Raw material 2	Component A	Component B
1	7	5	6	4
2	4	8	5	8
3	2	7	7	3

(b) Prove that a set S is convex *iff* every convex linear combination of points in S belongs to S.

(5+4)

Q2. (a) Consider the problem $Max\, z = -x_1 + 2x_2 - x_3$ subject to $x_1 + 2x_2 - 2x_3 \le 4$, $x_1 - x_3 \le 3$, $2x_1 - x_2 + 2x_3 \le 2$, $x_1, x_2, x_3 \ge 0$.

The optimal table of the above LPP is:

B.V	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z	9/2	0	0	3/2	0	1	8
x_2	3	1	0	1	0	1	6
s_2	7/2	0	0	½	1	1	7
x_3	5/2	0	1	½	0	1	4

(i) Find the range of the cost coefficient c_2 of variable x_2 such that present solution remains optimal.
(ii) If the RHS of the original problem is changed to (5,4,1) then find the optimal solution.
(iii) Find the optimal solution after adding a new constraint $3x_1 - x_2 \ge 1$.

(b) If X^* is a feasible solution of the primal problem Min $f(X) = C^T X$, $AX \ge b$, $X \ge 0$ and Y^* is a feasible solution of its dual Max $\varphi(Y) = b^T Y$, $A^T Y \le C$, $Y \ge 0$ such that $f(X^*) = \varphi(Y^*)$, then show that X^* and Y^* are optimal solution of primal and dual respectively.

((2+1+3) +3)

Q3. (a) Solve the following LPP using Revised Simplex Method:

$Max\; z = 4x_1 + 5x_2 - 3x_3$, subject to $x_1 + x_2 + x_3 = 10$, $x_1 - x_2 \ge 1$, $2x_1 + 3x_2 + x_3 \le 40$
$x_1, x_2, x_3 \ge 0$.

(b) Solve the following LPP using Graphical Method:

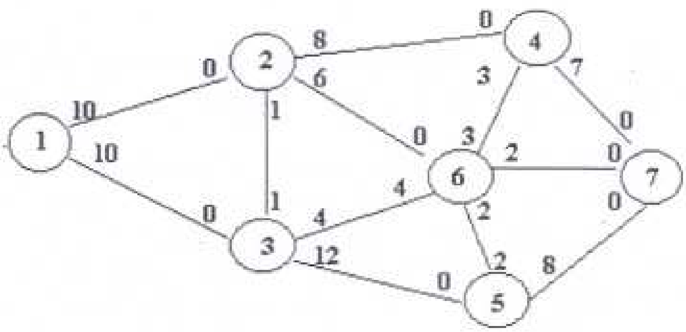
$Min\; z = 0.4x_1 + 0.8x_2$ subject to $x_1 + x_2 \ge 5$, $0.2x_1 - 0.35x_2 \le 0$, $0.02x_1 - 0.01x_2 \ge 0$,
$x_1 + 1.5x_2 \ge 6$, $x_1, x_2 \ge 0$.

(6+3)

Q4.(a) Use Branch and Bound method to solve following LPP:

$Min \;\; z = 5x_1 + 4x_2$ subject to $3x_1 + 2x_2 \geq 5$, $2x_1 + 3x_2 \geq 7$, $x_1, x_2 \geq 0$ and integer.

(b) Find maximum flow from Node '1' to '7' for the following network



(5+4)

Q5. (a) A company has team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of the salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below:

	1	2	3	4
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

Find the assignment(s) of salesmen to various districts which will yield maximum profit.

(b) Following table shows the activities and sequencing requirements necessary for the completion of a research project

Activity	Description	Predecessor(s)	Duration (weeks)
A	Literature search	--	6
B	Formulation of hypothesis	--	5
C	Preliminary feasibility study	B	2
D	Formal proposal	C	2
E	Field analysis	A,D	2
F	Progress report	D	1
G	Formal research	A,D	6
H	Data collection	E	5
I	Data analysis	G,H	6
J	Conclusions	I	2
K	Rough draft	G	4
L	Final copy	J,K	3
M	Preparation of oral presentation	L	1

(i) Draw a network diagram for this project.

(ii) Find the critical path and duration of the project.

(4+5)

Q6. (a) For Lagrange-Multiplier method, specify the necessary and sufficient conditions to get optimum of a non-linear programming problem. Also, solve the following problem using the same method:

$$Optimize\; z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 20$$

$$g(x) = x_1 + x_2 + x_3 = 11$$

(b) Solve the following problem(two iterations) using steepest descent method:

$Min \; f(x) = x_1^2 - x_1x_2 + 3x_2^2$, given that $X_1 = (1,2)^T$. (6+3)