

SCHOOL OF MATHEMATICS AND COMPUTER APPLICATIONS, T. I. E. T., Patiala

End Semester Examination- December 12, 2006

Max. Mark: 36

Numerical and Statistical Methods (MA-204)

Time: 03 Hours

Note: (a) Answer any FIVE questions by selecting any three from Group A and any two from Group B. (b) Write your tutorial group on the top of first page of answer sheet. (c) Attempt the parts of a problem at only one place.

SECTION-A

1. (a) Prove that the iterations generated by Bisection method always converge and its order of convergence is linear. (3.6)
- (b) Check whether the procedure to compute $f(x) = \sqrt{x+1} - \sqrt{x}$ at $x = 12345$ in six - decimal arithmetic is stable or unstable. Discuss the procedure if it is unstable. (3.6)

2. (a) Show that there are several second order Runge-Kutta methods and one of them is the improved Euler's method. (3.6)
- (b) The equation $x^3 - 7x^2 + 16x - 12 = 0$ has a double root at $x = 2$. Starting with initial approximation $x_0 = 1$, find the root correct to three decimal places using modified Newton-Raphson's method. (3.6)

3. (a) Let $p(x) = a_0 + (x - c_1)(a_1 + \dots + (x - c_n)(a_n) \dots)$ be a polynomial in Newton form. If $c_1 = c_2 = \dots = c_{r+1}$, then prove that $p^{(j)}(c_1) = j! a_j$, $j = 0, \dots, r$. (3.6)
- (b) Given that

$x :$	1.0	1.5	2.0	
$\log x :$	0.0	0.17609	0.30103	(3.6)

Find Lagrange interpolating polynomial with this data. If we add one more term $\log 3.0 = 0.47712$ in the above data then find $p_3(x)$ in such a way that $p_3(x) = p_2(x) + r(x)$. Find $r(x)$ explicitly. Also find the lower bound on interpolation error at $\bar{x} = 2.5$ for $p_3(x)$.

4. (a) Find the minimum number of intervals required to evaluate $\int_0^1 \frac{dx}{1+x}$ with accuracy 10^{-6} , by using the Simpson's one-third rule and also evaluate this integral using the same method. (3.0)
- (b) Using Power method, find the smallest eigenvalue and its corresponding eigenvector of the following matrix correct to two decimal places starting with initial approximation $X_0 = [1 \ 1]^T$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}. \quad (4.2)$$

SECTION-B

5. (a) The equations of two regression lines obtained in a regression analysis are as follows: $3X + 12Y = 19$, $3Y + 9X = 46$. Obtain (i) the value of correlation coefficient (ii) mean values of X and Y , and (iii) the ratio of the coefficient of variability of X to that of Y . (3.5)

(b) A continuous random variable X has the probability density function: $f(x) = A + Bx$, $0 \leq x \leq 1$. If the mean of the distribution is 0.5, find A and B . (1.5)

(c) A coin is flipped until 3 heads in succession occur. List only those elements of the sample space that require 6 or less tosses. Is this a discrete sample space? Explain. (2.2)

6. (a) The time it takes to repair a personal computer is random variable whose density function in hours, is given by $f(x) = \begin{cases} 1/2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ (2.5)

The cost of the repair depends on the time it takes and is equal to $40 + 30\sqrt{x}$ when the time is x . What is expected cost to repair a personal computer?

(b) Prove that the variance of normal distribution is σ^2 . (2.0)

(c) The probability of error in transmission of binary digit over a communication channel is 0.001. Write an expression for exact probability of more than 3 errors when transmitting a block of 1000 bits. What is its approximate value? Assume independent. (2.7)

7. (a) If U is uniformly distributed on $(0, 1)$ show that $a + (b - a)U$ is uniform on (a, b) . (3.0)

(b) A manufacturer produces bolts that are specified to be between 1.19 and 1.21 inches in diameter. If its production process results in a bolt's diameter being normally distributed with mean 1.20 inches a.s.d. 0.005. What percentage of bolt will not meet specification? (4.2)