**Thapar Institute of Engineering and Technology**
**Electronics and Communication Engineering Department**
**B.E. IIIrd Year, Final Examination**

Time: 3.0 Hrs. Marks: $12 \times 6 = 72$

*Attempt any six questions*

**Question 1.1:** Derive *Hilbert Transform Pair*. **(8)**

**Question 1.2:** What is *application of Hilbert Transformer* in the field of radar and speech signal processing? **(4)**

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**Question 2.1:** Consider an IIR filter transfer function

$$H[z] = \frac{1-2z^{-1}}{1+3z^{-1}}$$ **(3)**

Obtain its *Two-Band Decomposition* using polyphase decomposition technique.

**Question 2.2:** Design *a Decimator* for the reduction of sampling rate of a signal from $12KHz$ to $400Hz$ (as shown in Fig. 1) for the following specifications. **(2)**

Passband edge frequency $F_p = 180Hz$ Passband ripple $\delta_p = 0.002$

Stopband edge frequency $F_s = 200Hz$ Stopband ripple $\delta_s = 0.001$

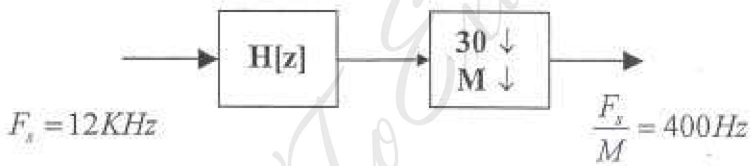


**Fig. 1:** Single-stage Decimator.

Design *Three-Stage Decimator* with **(7)**

$$M = M_1 \times M_2 \times M_3 = 5 \times 3 \times 2$$

How *Multistage Multirate Signal Processing* helps in reducing the computational complexity?

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**Question 3.0:** Consider *a Linear Predictor* of order two, described by

$$\hat{x}_n = h_1 x_{n-1} + h_2 x_{n-2}$$

where, the predictor coefficients $h_1$ and $h_2$ are selected to minimize the mean-square value of predictor error:

$$e_n = x_n - \hat{x}_n$$

**3.1)** Show that the optimum values of the predictor coefficients are given by

$$h_{o1} = \frac{\rho_1(1-\rho_2)}{1-\rho_1^2} \text{ and } h_{o2} = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}$$ **(6)**

$$\text{with } \rho_1 = \frac{R_X(1)}{R_X(0)} \text{ and } \rho_2 = \frac{R_X(2)}{R_X(0)}$$

**3.2)** Show that the minimum mean-square error equals

$$\xi_{\min} = \sigma_X^2 \left[ (1-\rho_1^2) - \frac{(\rho_1^2 - \rho_2)^2}{1-\rho_1^2} \right]$$ **(6)**

where, the variance $\sigma_X^2 = R_X(0)$.



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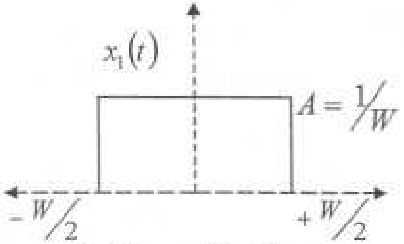
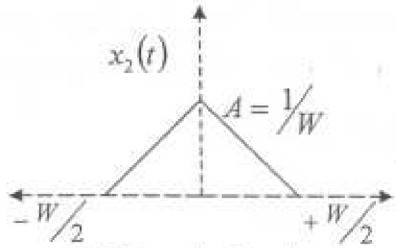
**Question 4.0:** Using derivative method,

**4.1)** Derive the *Continuous-Time Fourier Transform* of following **(6)**




(a) Square Function




(b) Triangular Function

**4.2)** Derive the *Discrete-Time Fourier Transform* of $y(n) = x(n) * u(n)$. **(4)**
where, $u(n)$ is the Step Function in discrete-domain.

**4.3)** What is relation between *correlation* and *convolution*? **(2)**

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**Question 5.1:** Determine the signal $x(n)$, whose *z-transform* is given by **(3)**

$$X[z] = \log(1 + az^{-1}) \qquad |z| > |a|$$

**Question 5.2:** Using z-transform, determine the *Auto-Correlation Sequence* of the signal

$$x(n) = a^n u(n), \qquad -1 < a < +1$$ **(4)**

**Question 5.3:** Describe the *Goertzel Algorithm* used to calculate DFT by the linear filtering approach. **(5)**

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**Question 6.1:** Prove that the *FIR filter* has a linear phase if its unit sample response satisfies the condition $h(n) = \pm h(M - n - 1)$ for $n = 0, 1, 2, \ldots, M-1$. **(4)**

**Question 6.2:** Describe the *Chirp-z Transform Algorithm* used to calculate DFT. **(4)**

**Question 6.3:** Design *a highpass filter* to meet the following specifications. **(4)**
Cutoff frequency $= 250Hz$, Sampling frequency $F_s = 1KHz$, and Filter length $= 7$.

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**Question 7.1:** Design *a lowpass filter* for the following specifications.

$$H_d(\omega) = \{e^{-j2\omega}, \; |\omega| \le \frac{\pi}{4} \quad \text{and} \quad H_d(\omega) = \{0, \; \frac{\pi}{4} \le |\omega| \le \pi$$ **(5)**

Use *rectangular window* $W_R(n) = 1, \quad 0 \le n \le 4$

**Question 7.2:** Use *Bilinear transformation* to design a highpass digital filter using *Butterworth approximation* to meet the following specifications. **(7)**

Stopband ripple $\le 15dB$
Passband edge $= 150Hz$
Passband attenuation $> 1dB$
Stopband edge $= 100Hz$
Sampling frequency $= 1KHz$