Thapar University , Patiala
B. Tech (Distance Education) Examination, September, 2007
Engineering Mathematics(MA 001D)

Time Allowed = 3 Hours **Maximum Marks : 100**

Note : All the questions all compulsory.

Q1. (a) Graph the following function. Discuss all the salient features also.

$$y = \frac{x^2 - 4}{x^2 - 2}.$$

(b) Identify the symmetry and sketch the following polar curve:

$$r = \sin\frac{\theta}{2}.$$

(8+8)

Q2. (a) For the following function, find the direction in which the function increase and decrease most rapidly at the given point P. Also find the derivative in these direction.

$$f(x, y, z) = xe^y + z^2 \qquad P(1, \ln 2, \frac{1}{2}).$$

(b) For the function $f(x, y) = x^2 + xy + y^2 - 6x$, find the absolute maxima and minima on the rectangular plate $0 \le x \le 5, \quad -3 \le y \le 3$.

(4+12)

Q3. (a) Find the Particular solution for following differential equation using operator method.

$$y'' - y' + y = x^3 - 3x^2 + 1.$$

(b) Find the complete solution of the following differential equation:

$$(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2$$

(6+10)

Q4. (a) Define row reduced echelon form and use it to examine whether the following system of equations is consistent or not, if yes find its solution

$$\begin{aligned} 2x_1 + x_2 + 2x_3 &= 1 \\ x_1 + x_2 &= 0 \\ x_1 - 2x_2 + 6x_3 &= 3 \\ x_1 - 2x_3 &= 1 \\ x_1 - x_2 + 4x_3 &= 2 \end{aligned}$$

(b) Find the inverse of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ **(10+6)**

Q5. (a) State and prove convolution theorem and hence find $L^{-1}\left[\frac{1}{s^2(s^2+1)}\right]$

(b) Find the Fourier series of the function $f(x) = x + x^2, \quad -\pi < x < \pi$

(8+8)

Q6. (i) (a) Examine the function $f(x) = x^3 - 3x + 3,\ x \in \Re$ and

(b) $f(x) = \sin^2 x,\ 0 < x < \pi$ for maximum and minimum values.

(ii) The function $f(x) = \begin{cases} x & 0 \le x < 1 \\ 0 & x = 1 \end{cases}$ is zero at $x = 0$ and $x = 1$ and differentiable on (0, 1), but its derivative on (0, 1) is never zero. How can this be? Doesn't Rolle's theorem say the derivative has to be zero somewhere in (0,1)? Give reasons for your answer.

(iii) Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, -5) if $f(x, y) = x^2 + 3xy + y - 1$.

(iv) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^3 + y^3;\ x = r - 2s;\ y = r + 2s$.

(v) Solve the following first order differential equation: $dx + (3x - e^{-2y})dy = 0$.

(vi) Solve the following Euler-Cauchy equation : $2x^3y'' + 10xy' + 8y = 0$.

(vii) Find $L\left[t^3 e^{5t}\right]$.

(viii) Find $L\left[e^{t-4}u(t-4)\right]$

(ix) Determine the eigen values of $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$

(x) Write the matrix $A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 8 & 4 \\ 2 & 6 & 10 \end{bmatrix}$ as the sum of symmetric and skew-symmetric matrix.

(2 x 10)