Computer Applications Paper
2007

Special Instructions / Useful Data

$\mathbf{N}$ denotes the set of natural numbers.

$\mathbf{Z}$ denotes the set of integers.

$\mathbf{Q}$ denotes the set of rational numbers.

$\mathbf{R}$ denotes the set of real numbers.

P^T denotes the transpose of a matrix P.

$\overline{x}$ denotes the complement of a Boolean variable x.

f' denotes the derivative of a function f.

$\frac{\partial z}{\partial x}$ denotes the partial derivative of z with respect to x.

$E(X)$ denotes the expected value of a random variable X.

$\mathbf{R}^3 = \{\mathbf{x}^T = (x_1, x_2, x_3) : x_1, x_2, x_3 \in \mathbf{R}\}$

$\mathbf{Z}_n = \{0, 1, 2, \cdots, n-1\}$

$\oplus_n$ denotes addition modulo n.

All **bold faced** vectors are column vectors.

For all **C programs** assume that all standard library functions are accessible.

1. Let

$$F(x) = \int_0^x (t-1)(t-2)(t-3)(t-4)\,dt, \quad 0 \le x \le 5.$$

Then F has local minimum at the points

(A) $\{0, 2, 4\}$

(B) $\{1, 3, 5\}$

(C) $\{0, 3, 4\}$

(D) $\{3, 4, 5\}$

2. The integral

$$\int_0^{\frac{\pi}{2}} \min(\sin x, \cos x)\,dx$$

equals

(A) $\sqrt{2} - 2$

(B) $2 - \sqrt{2}$

(C) $2\sqrt{2}$

(D) $2 + \sqrt{2}$

3. For $n \ge 5$, the expression

$$1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1}, \quad x \ne 1,$$

is equal to

(A) $\dfrac{nx^n(1-x) - x^n + 1}{(1-x)^2}$

(B) $\dfrac{nx^n(x-1) - x^n + 1}{(1-x)^2}$

(C) $\dfrac{nx^n(x-1) + x^n - 1}{(1-x)^2}$

(D) $\dfrac{nx^n}{(1-x)^2}$

4. The function f defined on $\mathbb{R}$ by

$$f(x) = 3^x + 4^x - 5^x$$

has

(A) exactly one zero

(B) exactly two zeros

(C) exactly three zeros

(D) infinitely many zeros

5. The spheres

$$x^2 + y^2 + z^2 = 1 \text{ and } x^2 + \left(y - \sqrt{3}\right)^2 + z^2 = 4$$

intersect at an angle

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

6. If Ω denotes the region bounded by the X-axis and the lines $y = x$ and $x = 1$, then the value of the integral

$$\iint_{\Omega} \frac{\cos(2x)}{x} dx\, dy$$

is

(A) $\frac{\sin 2}{2}$

(B) $\frac{\cos 2}{2}$

(C) $\cos 2$

(D) $\sin 2$

7. Let f be an increasing, differentiable function. If the curve $y = f(x)$ passes through $(1, 1)$ and has length

$$L = \int_1^2 \sqrt{\left(1 + \frac{1}{4x^2}\right)}\, dx,\ 1 \le x \le 2,$$

then the curve is

(A) $y = \ln\left(\sqrt{x}\right) - 1$

(B) $y = 1 - \ln\left(\sqrt{x}\right)$

(C) $y = \ln\left(1 + \sqrt{x}\right)$

(D) $y = 1 + \ln\left(\sqrt{x}\right)$

8. If the line $y = mx,\ 0 \le x \le 2$ is rotated about the line $y = -1$, then the area of the generated surface is

(A) $4\pi(1+m)\sqrt{1+m}$

(B) $4\pi\left(1+m^2\right)\sqrt{1+m}$

(C) $4\pi\left(1+\sqrt{m}\right)\sqrt{1+m^2}$

(D) $4\pi(1+m)\sqrt{1+m^2}$

9. Let D be the region in the first quadrant lying between $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. The value of the integral

$$\iint_D \sin\left(x^2 + y^2\right) dx\, dy$$

is

(A) $\frac{\pi}{4}(\cos 1 - \cos 2)$

(B) $\frac{\pi}{4}(\cos 1 - \cos 4)$

(C) $\frac{\pi}{2}(\cos 1 - \cos 2)$

(D) $\frac{\pi}{2}(\cos 1 - \cos 4)$

10. Consider the function

$$f(x, y) = (x+y)^2 - (x+y) + 1 .$$

The absolute maximum value and the absolute minimum value of the function on the unit square $\{(x, y): 0 \le x \le 1,\ 0 \le y \le 1\}$, respectively are

(A) 3 and $\frac{3}{2}$

(B) $\frac{3}{2}$ and $\frac{3}{4}$

(C) 3 and $\frac{3}{4}$

(D) 2 and $\frac{3}{4}$

11. For $\lambda > 0$, the value of the integral

$$\int_0^\infty e^{-\lambda x^2}\, dx$$

equals

(A) $\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$

(B) $\sqrt{\frac{\pi}{2\lambda}}$

(C) $\sqrt{\frac{2\pi}{\lambda}}$

(D) $2\sqrt{\frac{\pi}{\lambda}}$

12. Suppose

$$z = x\sin\left(\frac{x}{y}\right) + y\sin\left(\frac{y}{x}\right),\ xy \neq 0.$$

Then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(A) $-z$

(B) 0

(C) z

(D) $2z$

13. Let

$$f(x) = x^3 - x^2 + 1,\ 0 \leq x \leq 1.$$

Then the absolute minimum value of $f(x)$ is

(A) $\frac{14}{27}$

(B) $\frac{5}{9}$

(C) $\frac{23}{27}$

(D) 1

14. The slope of the tangent line to the curve

$$x = a(t - \sin t),\ y = a(1 - \cos t), t \in \mathbb{R},$$

at $t = \frac{\pi}{2}$ is

(A) -1

(B) 0

(C) 1

(D) ∞

15. Consider the equations

$$\sin(\cos x) = x \qquad (1)$$

and

$$\cos(\sin x) = -x \qquad (2)$$

for $x \geq 0$. Then

(A) Only Equation (1) has a solution

(B) Only Equation (2) has a solution

(C) Both Equations (1) and (2) have solutions

(D) Neither Equation (1) nor Equation (2) has a solution

16. If

$$\lim_{h \to 0} \frac{1}{h} \int_{\alpha}^{\alpha+h} e^{-t^2} dt = 1,$$

then the value of α is

(A) -1

(B) 0

(C) 1

(D) 2

17. Let

$$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 5.$$

Then the local maximum and the local minimum of the function f are at the points

(A) $(-2, 0)$ and $(-2, 2)$, respectively

(B) $(-2, 0)$ and $(0, 2)$, respectively

(C) $(0, 2)$ and $(-2, 0)$, respectively

(D) $(0, 2)$ and $(0, 0)$, respectively

18. Let θ, $0 \le \theta \le \pi$ be the angle between the planes

$$x - y + z = 3 \text{ and } 2x - z = 4.$$

The value of θ is

(A) $\cos^{-1}\left(\frac{1}{5}\right)$

(B) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{15}}\right)$

(D) $\cos^{-1}\left(\frac{3}{\sqrt{15}}\right)$

19. Let

$$f(x, y) = xy^2 + yx^2.$$

Suppose the directional derivative of f in the direction of the unit vector (u_1, u_2) at the point $(1, -1)$ is 1. Then among the following, (u_1, u_2) is

(A) $(-1, 0)$

(B) $(0, 1)$

(C) $(1, 0)$

(D) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

20. The point on the sphere $x^2+y^2+z^2=1$ farthest from the point $(1,-2,1)$ is

(A) $\left(\frac{-1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{-1}{\sqrt{6}}\right)$

(B) $\left(\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$

(C) $\left(\frac{-1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$

(D) $\left(\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{-1}{\sqrt{6}}\right)$

21. Consider the dihedral group $D_4=\{e,r,r^2,r^3,f,rf,r^2f,r^3f\}$ with $r^4=e=f^2$ and $rf=fr^{-1}$. The product $r^3fr^{-1}f^{-1}r^3fr$ corresponds to

(A) f

(B) rf

(C) r^2f

(D) r^3f

22. Let p and q be distinct primes and H be a proper subgroup of the additive group of integers. Suppose $S=H\cap\{p,q,p+q,pq,p^q,q^p\}$ has exactly three elements. Then S is

(A) $\{pq,p^q,q^p\}$

(B) $\{p+q,pq,p^q\}$

(C) (p,pq,p^q)

(D) $\{p,p+q,pq\}$

23. The number of group homomorphisms from the group $(Z_{18}, \oplus_{18})$ to the group $(Z_{30}, \oplus_{30})$ is

(A) 3

(B) 4

(C) 5

(D) 6

24. Let $\sigma = (125)(36)$ and $\tau = (1456)(23)$ be two elements of the permutation group on 6 symbols. Then the product $\sigma \circ \tau$, where $\sigma \circ \tau(i) = \sigma(\tau(i))$, is

(A) $(14)(26)(35)$

(B) $(13)(26)(45)$

(C) $(14)(25)(36)$

(D) $(13)(24)(56)$

25. Let $G = \{n \in Z : 1 \leq n \leq 55, \text{ gcd } (n, 56) = 1\}$ be a multiplicative group modulo 56. Consider the sets

$$S_1 = \{1, 9, 17, 25, 33, 41\} \text{ and } S_2 = \{1, 15, 29, 43\}.$$

Which one of the following is TRUE?

(A) S_1 is a subgroup of G but S_2 is NOT a subgroup of G

(B) S_1 is NOT a subgroup of G but S_2 is a subgroup of G

(C) Both S_1 and S_2 are subgroups of G

(D) Neither S_1 nor S_2 is a subgroup of G

26. Let G be a group with respect to multiplication. If $x = \alpha\sqrt{2} + \beta\sqrt{3} \in G$ then x^{-1} is

(A) $\dfrac{\alpha\sqrt{2} + \beta\sqrt{3}}{2\alpha^2 + 3\beta^2}$

(B) $\dfrac{\alpha\sqrt{2} - \beta\sqrt{3}}{2\alpha^2 - 3\beta^2}$

(C) $\dfrac{\alpha\sqrt{2} + \beta\sqrt{3}}{2\alpha^2 - 3\beta^2}$

(D) $\dfrac{\alpha\sqrt{2} - \beta\sqrt{3}}{2\alpha^2 + 3\beta^2}$

27. Let $G = \{1, 2, \cdots, p-1\}$ be the group with respect to multiplication modulo p. If the inverse of 110 in G is 4, then p is of the form

(A) $5n + 1$

(B) $5n + 2$

(C) $5n + 3$

(D) $5n + 4$

28. Consider the alternating group $A_4 = \{\sigma \in S_4 : \sigma \text{ is an even permutation}\}$. Which of the following is FALSE?

(A) A_4 has 12 elements

(B) A_4 has exactly one subgroup of order 4

(C) A_4 has a subgroup of order 6

(D) Number of 3 cycles in A_4 is 8

29. Let P be a 3×3 matrix such that for some $\mathbf{c}$, the linear system $P\mathbf{x}=\mathbf{c}$ has infinite number of solutions. Which one of the following is TRUE?

(A) The linear system $P\mathbf{x}=\mathbf{b}$ has infinite number of solutions for all $\mathbf{b}$

(B) Rank $(P)=3$

(C) Rank $(P)\neq1$

(D) Rank $(P)\leq2$

30. Let

$$f(x)=x^3+x^2-x+15 \text{ and } g(x)=x^3+2x^2-x+15.$$

Then, over $\mathbb{Q}$

(A) f is irreducible and g is reducible

(B) f is reducible and g is irreducible

(C) Both f and g are reducible

(D) Both f and g are irreducible

31. Let

$$U=\left\{\begin{pmatrix}1&a&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}:a\in\mathbb{R}\right\} \text{ and } V=\left\{\begin{pmatrix}a&0&0\\0&a&0\\0&0&1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}:a\in\mathbb{R}\right\}.$$

The angle between U and V is

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

32. Let

$$P = \begin{pmatrix} 1 & 0 & 5 \\ 1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix}.$$

Then $8P^{-1}$ is equal to

(A) $\begin{pmatrix} 13 & -4 & -1 \\ -15 & 4 & 3 \\ 10 & 0 & -2 \end{pmatrix}$

(B) $\begin{pmatrix} 13 & -15 & 10 \\ -4 & 4 & 0 \\ -1 & 3 & -2 \end{pmatrix}$

(C) $\begin{pmatrix} 13 & 10 & -15 \\ -4 & 0 & 4 \\ -1 & -2 & 3 \end{pmatrix}$

(D) $\begin{pmatrix} 13 & -4 & -1 \\ 10 & 0 & -2 \\ -15 & 4 & 3 \end{pmatrix}$

33. Let P be an $n \times n$ idempotent matrix, that is, $P^2 = P$. Which of the following is FALSE?

(A) P^T is idempotent

(B) The possible eigenvalues of P are 0 or 1

(C) The nondiagonal entries of P can be zero

(D) There are infinite number of $n \times n$ nonsingular matrices that are idempotent

34. Let V be the vector space of all polynomials with real coefficients. If W is the vector subspace of V generated by

$$1-x,\ x^2-x,\ x^2-1 \text{ and } x^2-3x+2,$$

then the dimension of W is

(A) 1

(B) 2

(C) 3

(D) 4

35. Let

$$P=\begin{pmatrix}1 & i\\ i & -1\end{pmatrix}.$$

Then

(A) P has two linearly independent eigenvectors

(B) P has an eigenvector

(C) P is nonsingular

(D) There exists a nonsingular matrix S such that $S^{-1}PS$ is a diagonal matrix

36. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $\mathbf{v} \neq \mathbf{0}$. Which of the following is FALSE?

(A) $\left|\mathbf{u}\cdot\frac{\mathbf{v}}{\|\mathbf{v}\|}\right|$ is the length of the projection of $\mathbf{u}$ along $\mathbf{v}$

(B) If $\mathbf{u}\cdot\mathbf{w}=\mathbf{v}\cdot\mathbf{w}$ for all $\mathbf{w}\in\mathbb{R}^3$, then $\mathbf{u}=\mathbf{v}$

(C) $\mathbf{u}\cdot\mathbf{v}=\frac{1}{2}\left(\|\mathbf{u}+\mathbf{v}\|^2-\|\mathbf{u}-\mathbf{v}\|^2\right)$

(D) $\|\mathbf{u}+\mathbf{v}\|^2+\|\mathbf{u}-\mathbf{v}\|^2=2\left(\|\mathbf{u}\|^2+\|\mathbf{v}\|^2\right)$

37. Let P be a 2×2 matrix such that $P^{102} = \mathbf{0}$. Then

(A) $P^2 = \mathbf{0}$

(B) $(I-P)^2 = \mathbf{0}$

(C) $(I+P)^2 = \mathbf{0}$

(D) $P = \mathbf{0}$

38. Let

$$P = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The eigenvectors corresponding to the eigenvalues i and $-i$ are respectively

(A) $\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\begin{pmatrix} -1 \\ i \end{pmatrix}$

(B) $\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\begin{pmatrix} i \\ -i \end{pmatrix}$

(C) $\begin{pmatrix} -1 \\ i \end{pmatrix}$ and $\begin{pmatrix} i \\ -i \end{pmatrix}$

(D) $\begin{pmatrix} i \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ i \end{pmatrix}$

39. The area of the parallelogram with sides

$$\mathbf{x} = \vec{i} + \vec{j} + \vec{k} \text{ and } \mathbf{y} = -\vec{i} + \vec{j}$$

is

(A) $\sqrt{6}$

(B) $2\sqrt{3}$

(C) $3\sqrt{2}$

(D) 6

40. Let

$$\mathbf{x} = \vec{i} + \vec{j} + \vec{k},\ \mathbf{y} = \alpha\vec{i} + \vec{k} \text{ and } \mathbf{z} = \vec{i} + \alpha\vec{j}.$$

Then the volume of the parallelopiped with sides $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ is

(A) $1 + \alpha + \alpha^2$

(B) $1 + \alpha - \alpha^2$

(C) $1 - \alpha + \alpha^2$

(D) $\alpha^2 + \alpha - 1$

41. The solution of the initial value problem

$$xy' - y = 0$$

with $y(1) = 1$ is

(A) $y(x) = x$

(B) $y(x) = \frac{1}{x}$

(C) $y(x) = 2x - 1$

(D) $y(x) = \frac{1}{2x - 1}$

42. Let $y(x) = x \sin x$ be one of the solution of an n^{th} order linear differential equation with constant coefficients. Then the minimum value of n is

(A) 1

(B) 2

(C) 3

(D) 4

43. The solution of the differential equation

$$\left(x^2y+xy^2\right)dx+\left(\frac{x^3}{3}+x^2y+\sin y\right)dy=0$$

is

(A) $\frac{x^3y}{3}+\frac{x^2y^2}{2}-\cos y=c$

(B) $\frac{x^3y}{3}+\frac{x^2y^2}{2}+\cos y=c$

(C) $\frac{x^3}{3}+\frac{x^2y^3}{6}-\cos y=c$

(D) $\frac{x^3}{3}+\frac{x^2y^3}{6}+\cos y=c$

44. The general solution of the differential equation

$$y'''+y''-y'-y=0$$

is

(A) $\left(c_1+xc_2+x^2c_3\right)e^x$

(B) $\left(c_1+xc_2+x^2c_3\right)e^{-x}$

(C) $c_1e^x+\left(c_2+xc_3\right)e^{-x}$

(D) $\left(c_1+xc_2\right)e^x+c_3e^x$

45. Let

$$f(x) = 2x^3 - x^2 + 2x - 5 .$$

Consider the following statements about the roots of $f(x) = 0$

P : At least one root is positive.

Q : At least one root is negative.

R : There is a root between $x = 1$ and $x = 2$.

Which one of the following is TRUE?

(A) P, Q and R are valid statements

(B) P and Q are valid statements but R is NOT a valid statement

(C) P and R are valid statements but Q is NOT a valid statement

(D) P is a valid statement but Q and R are NOT valid statements

46. The maximum absolute error that occurs in rounding off a number after 6 places of decimal is

(A) 5×10^{-8}

(B) 10^{-7}

(C) 5×10^{-7}

(D) 5×10^{-6}

47. Which of the following is FALSE?

(A) A unique interpolating polynomial of degree n is obtained from the given values at fixed $n+1$ points

(B) The Lagrange interpolation formula can be applied to equispaced points

(C) The Newton's forward difference interpolation formula can be applied to non-equispaced points

(D) The trapezoidal rule gives exact value of the integral for linear functions

48. Consider

x	-1	0	1	2	3
$f(x)$	1	5	3	1	5

Applying Simpson's one third rule, the value of the integral

$$\int_{-1}^{3} f(x)\,dx$$

is

(A) 10

(B) 12

(C) $\frac{41}{3}$

(D) 15

49. Consider

$$f(x) = 1 + xe^{-x}.$$

The Newton-Raphson iterative scheme for finding a root of $f(x) = 0$ is

(A) $x_{n+1} = \dfrac{1 + x_n^2 e^{-x_n}}{(x_n - 1)e^{-x_n}}$

(B) $x_{n+1} = \dfrac{x_n^2 e^{-x_n} + x_n\left(1 + e^{-x_n}\right) - 1}{1 + x_n e^{-x_n}}$

(C) $x_{n+1} = \dfrac{x_n^2 e^{-x_n} + x_n\left(1 - e^{-x_n}\right) + 1}{1 + x_n e^{-x_n}}$

(D) $x_{n+1} = \dfrac{1 + x_n^2 e^{-x_n}}{1 + x_n e^{-x_n}}$

50. Consider the following Primal Linear Programming Problem :

Maximize $\mathbf{c}^T\mathbf{x}$

Subject to $P\mathbf{x} = \mathbf{b}$

$\mathbf{x} \geq \mathbf{0}$

The Dual Linear Programming Problem is

(A) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} = \mathbf{c}$, $\mathbf{y}$ unrestricted

(B) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} \geq \mathbf{c}$, $\mathbf{y}$ unrestricted

(C) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} = \mathbf{c}, \mathbf{y} \geq \mathbf{0}$

(D) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} \geq \mathbf{c}, \mathbf{y} \geq \mathbf{0}$

51. If the Primal Linear Programming Problem is unbounded then which of the following is TRUE?

(A) Dual problem is unbounded

(B) Dual problem has a single bounded optimal solution

(C) Dual problem has multiple bounded optimal solutions

(D) Dual problem is infeasible

52. Which of the following pair of linear programming constraints is equivalent to the inequality $|x_1 - x_2| \leq a$?

(A) $x_1 - x_2 \leq a,\ x_2 - x_1 \leq a$

(B) $x_1 - x_2 \leq a, x_2 - x_1 \leq -a$

(C) $x_1 - x_2 \leq -a,\ x_2 - x_1 \leq -a$

(D) $x_1 - x_2 \leq -a,\ x_2 - x_1 \leq a$

53. Consider the following Linear Programming Problem :

Maximize $3x_1 + 8x_2$

Subject to $2x_1 + 5x_2 \leq 10$

$6x_1 + x_2 \leq 6$

$x_1, x_2 \geq 0$

The optimal value of the objective function is

(A) 0

(B) 3

(C) $\frac{111}{7}$

(D) 16

54. A cow is tied with a pole by a 100 meter long rope. What is the probability that at some point of time the cow is at least 60 meters away from the pole?

(A) $\frac{9}{25}$

(B) $\frac{13}{25}$

(C) $\frac{16}{25}$

(D) $\frac{18}{25}$

55. Two letters are chosen one after another without replacement from the English alphabet. What is the probability that the second letter chosen is a vowel?

(A) $\frac{4}{25}$

(B) $\frac{5}{26}$

(C) $\frac{5}{25}$

(D) $\frac{1}{5} \cdot \frac{1}{26}$

56. Let X be a binomial random variable with parameters n and p. If the mean and the standard deviation of X are 3 and $\frac{3}{2}$, respectively, then what is the value of (n, p)?

(A) $\left(4, \frac{3}{4}\right)$

(B) $\left(6, \frac{1}{2}\right)$

(C) $\left(9, \frac{1}{3}\right)$

(D) $\left(12, \frac{1}{4}\right)$

57. Let X, Y, Z be independent Poisson variables, such that $E(X) = E(Y)$ and $E(Z) = 2E(X)$. If $P(X = 5, Y = 4)$ is equal to $P(Z = 8)$, then $E(X)$ is

(A) $\frac{2^6}{21}$

(B) $\frac{2^6}{7}$

(C) $\frac{2^7}{7}$

(D) $\frac{2^7}{21}$

58. The largest natural number whose base 7 representation has exactly four digits, is

(A) 2400

(B) 6666

(C) 7777

(D) 2401

59. 10's complement of the decimal number 56789 is

(A) 01234

(B) 12345

(C) 43210

(D) 43211

60. Let $x = 0.125E + 01$, $y = (1.01)_2$ and $z = (1.2)_8$. Which of the following is TRUE?

(A) x, y and z are equal

(B) Only x and y are equal

(C) Only x and z are equal

(D) All x, y and z are different

61. The decimal value of $(21)_8 \times (101)_{16}$ lies in the interval

(A) 3000 – 3499

(B) 3500 – 3999

(C) 4000 – 4499

(D) 4500 – 4999

62. The binary equivalent of the hexadecimal number $A52C$ is

(A) 1010101101100

(B) 1010010100101100

(C) 1010111000101100

(D) 1010010100101101

63. Let x, y and z be Boolean variables. The number of possible values for the expression

$$xy + \bar{z}\,x$$

is

(A) 1

(B) 2

(C) 4

(D) 8

64. Let x and y be independent Boolean variables, each taking values 0 or 1 with probabilities 0.5 and 0.5, respectively. The probability that

$$x + y(\bar{x} + \bar{y}) = 1$$

is

(A) 0

(B) 0.25

(C) 0.5

(D) 0.75

65. The Boolean expression

$$(x + y)(y + \bar{z})(z + \bar{x})$$

is equal to

(A) xyz

(B) $xy\bar{z}$

(C) $(\bar{x} + z)y$

(D) $(x + \bar{z})y$

66. Let X and Y be 4 bit registers with initial contents as 1011 and 1001, respectively. The following sequence of operations are performed on the two registers :

$$Y \leftarrow X \oplus Y$$
$$X \leftarrow X \oplus Y$$
$$Y \leftarrow X \oplus Y$$

where $\oplus$ denotes *XOR* operation. The final contents of the two registers are

(A) $X = 1001, Y = 1011$

(B) $X = 1011, Y = 1001$

(C) $X = 1011, Y = 1011$

(D) $X = 1001, Y = 1001$

67.

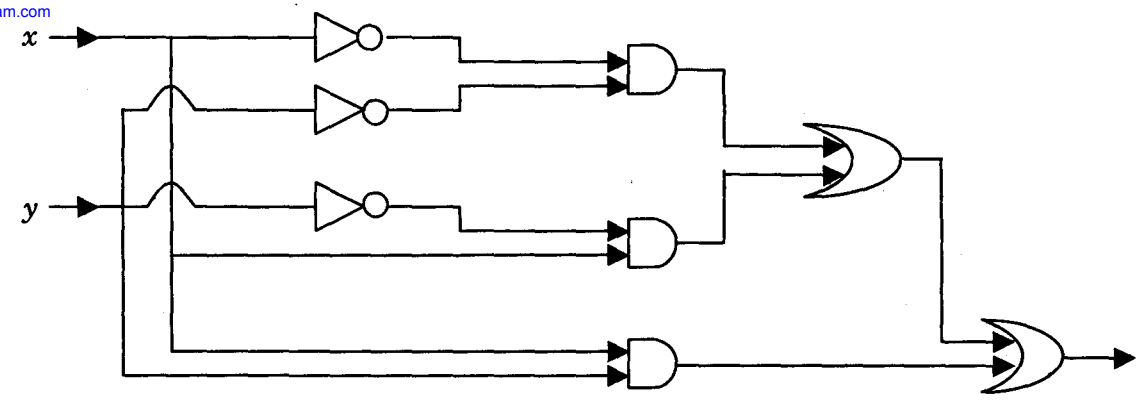

Figure 1

The logic circuit diagram shown in Figure 1 is equivalent to the Boolean expression

(A) $x + y$

(B) $x + \overline{y}$

(C) $\overline{x} + y$

(D) $\overline{x} + \overline{y}$

68.

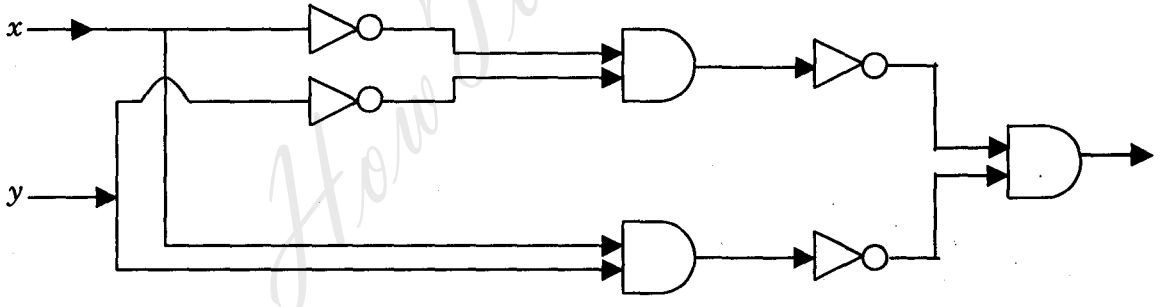

Figure 2

The logic circuit diagram given in Figure 2 is equivalent to

(A) AND gate

(B) OR gate

(C) NAND gate

(D) XOR gate

69. BIOS is the acronym for

(A) Binary Input Output Source

(B) Basic Input Output Support

(C) Binary Input Output System

(D) Basic Input Output System

70. The maximum number of characters that can be encoded in a fixed length encoding schem with n bits is

(A) 2^n

(B) $n!$

(C) n^2

(D) n

71. Which of the following is an 8-bit processor?

(A) Intel 80286

(B) Intel 8086

(C) Intel 8085

(D) Intel Pentium II

72. For which of the following combinations an SR Flip-Flop is set to 1?

(A) S = 0, R = 0

(B) S = 0, R = 1

(C) S = 1, R = 0

(D) S = 1, R = 1

73. For which of the following combinations, a JK Flip-Flop will enter into the complement of the present state?

(A) J = 0, K = 0

(B) J = 0, K = 1

(C) J = 1, K = 0

(D) J = 1, K = 1

74. Which of the following is NOT a Software?

(A) Adobe

(B) Browser

(C) Compiler

(D) Device Driver

75. Match the items of **List 1** with the items of **List 2**

List 1		List 2	
1.	Operating Systems	P.	Pentium
2.	Application Software	Q.	Linux
3.	Processor	R.	Router
4.	Network	S.	Anti Virus

(A) (1, Q), (2, S), (3, P), (4, R)

(B) (1, Q), (2, R), (3, P), (4, S)

(C) (1, P), (2, S), (3, Q), (4, R)

(D) (1, P), (2, R), (3, S), (4, Q)

76. Match the file extensions in **List 1** with the corresponding applications in **List 2**

List 1		List 2	
1.	mp3	P.	image
2.	xls	Q.	music
3.	jpeg	R.	database
4.	mdb	S.	spread sheet

(A) (1, Q), (2, S), (3, R), (4, P)

(B) (1, Q), (2, S), (3, P), (4, R)

(C) (1, Q), (2, P), (3, S), (4, R)

(D) (1, Q), (2, R), (3, P), (4, S)

77. Which of the following is NOT a Random Access Storage Device?

(A) Magnetic Tape

(B) Hard Disk

(C) Floppy Disk

(D) CD

78. Which of the following is a valid C directive?

(A) # include <stdio.h>;

(B) # include <stdio.h>

(C) include <stdio.h>;

(D) include <stdio.h>

79. Consider the following declaration in C

```
struct student {
        char name[12] ;
        float gradepoint ;
};
struct student MCA[5] ;
```

The number of bytes needed to store the array *MCA* is

(A) 16

(B) 25

(C) 70

(D) 80

80. Consider the following C statements

P : for $(i = 0; i < 8; i+ = 3)$ {printf ("*");}

Q : for $(i = 4; i > 0; i- = 2)$ {printf ("*");}

R : for $(i = 0; i <= 9; i+ = 3)$ {printf ("*");}

S : for $(i = 0; i < 7; i++)$ {if $(i\%3 == 0)$ printf ("*");}

Which one of the following is a TRUE statement?

(A) P, Q, R and S give the same output

(B) P and S give the same output

(C) Q and R give the same output

(D) P, Q and S give the same output

81. Consider the following program segment

```
{ int x, i, j ;
    x = 0 ;
    for (i = 0; i < 19; i++)
      for (j = i + 1; j < 20; j++)
        x++; }
```

The value of x after executing the segment is

(A) 171

(B) 190

(C) 342

(D) 380

82. Let $f: \mathbf{N} \to \mathbf{N}$ be defined as

$$f(n) = \begin{cases} 1, & \text{if } n = 1 \text{ or } n = 2 \\ f(n-1) + f(n-2), & \text{otherwise.} \end{cases}$$

What is the value of $f(10)$?

(A) 34

(B) 45

(C) 55

(D) 89

83. Consider the following program segment

```
{ int n = 1;
  float x, term;
  float sum = 1;
  term = 1;
  while (n < 51)
  {
      term * = -x * x/(n * (n + 1));
      sum + = term ;
      n + = 2;
  }
}
```

For a given x the value of *sum* approximates the function

(A) $\sin x$

(B) $\cos x$

(C) e^{-x}

(D) e^{-x^2}

84. Consider the following program

```
void swap (int a, int b)
{ int temp ;
 temp = a ;
a = b ;
b = a ;
}
void main ( )
{ int x, y;
  x = 2; y = 3;
  swap (x, y);
  printf ("x = %d y = %d \n", x, y);
}
```

The output of the program is

(A) $x = 2 \quad y = 2$

(B) $x = 2 \quad y = 3$

(C) $x = 3 \quad y = 2$

(D) $x = 3 \quad y = 3$

85. What is the output of the following C program?

```
void fun (int * p)
{ int i, sum = 0 ;
  for (i = 2; i < 4; ++ i)
        sum + = * (p + i);
  printf ("%d", sum);
}
 void main ( )
{ int a[5] = {10, 20, 30, 40, 50};
  fun (a + 1);
}
```

(A) 90

(B) 120

(C) 130

(D) 140

86. Consider the following C program segment

```
int gradepoint ;
char ch;
switch (ch) {
   case 'A' :   {gradepoint = 10 ;}
   case 'B' :   {gradepoint = 8 ; break ; }
   case 'C' :   {gradepoint = 6 ; }
   default :    {gradepoint = 0 ; } }
```

Executing the program segment for *ch* = 'A', 'B', 'C' *gradepoints* are respectively

(A) 10, 8, 6

(B) 10, 8, 0

(C) 8, 8, 6

(D) 8, 8, 0

87. Consider the following C program

```
void main ( )
{
   int i, s ;
   for (i = 0; ; i + +)
    { s = s + i / (i – 2) ;
      if (i > 5) break ;
    }
}
```

Which one of the following is a TRUE statement?

(A) There is a syntax error

(B) There is a type mismatch error

(C) There is a runtime error

(D) There is no runtime error

88. The unit place of the number 27^{82} is

(A) 1

(B) 3

(C) 7

(D) 9

89. The number of all functions $f:\{1,2,\cdots,n\}\to\{1,2,\cdots,m\}$ is

(A) $m(m-1)\cdots(m-n+1)$

(B) $n(n-1)\cdots(n-m+1)$

(C) m^n

(D) n^m

90. The number of ways in which 4 boys and 5 girls can sit in a row so that there is a girl between any two boys is

(A) 4! 5!

(B) 3 (4! 5!)

(C) 5 (4! 5!)

(D) 15 (4! 5!)

91. The next term in the series 191, 211, 232, 254, ---- is

(A) 267

(B) 276

(C) 277

(D) 287

92. If $\sin x+\cos x=\alpha$ then $\sin(2x)$ is

(A) $1-\alpha^2$

(B) α^2-1

(C) $1+\alpha^2$

(D) α^2

93. A student computes the sum of squares of the first 40 natural numbers and gives incorrect answer 22019. By mistake, the student forgot to add the square of one of t numbers. The missed number is

(A) 5

(B) 7

(C) 9

(D) 11

94. For $a, b \in \mathbf{Z}$, define a relation aRb if $ab \geq 0$. Then the relation R is

(A) symmetric, reflexive and transitive

(B) symmetric and reflexive but NOT transitive

(C) symmetric and transitive but NOT reflexive

(D) reflexive and transitive but NOT symmetric

95. What is the sum of the interior angles of an n vertex simple polygon?

(A) $(n-2)\pi$

(B) $\frac{(n+3)\pi}{6}$

(C) $\frac{(n+1)\pi}{4}$

(D) $\frac{n\pi}{3}$

96. Who among the following is NOT a Nobel Laureate?

(A) Amartya Sen

(B) J.C. Bose

(C) Muhammad Yunus

(D) S. Chandrasekhar

97. Who is the father of Bhishma in the Mahabharata?

(A) Bharat

(B) Devavrata

(C) Parashar

(D) Shantanu

98. Which country won the 2006 FIFA World Cup?

(A) Argentina

(B) France

(C) Germany

(D) Italy

99. Which of the following diseases is NOT caused by mosquito bite?

(A) Dengue

(B) Encephalitis

(C) Malaria

(D) Typhoid

100. The value of

$$3+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ddots}}}}$$

is

(A) $2-\sqrt{2}$

(B) $3-\sqrt{2}$

(C) $2+\sqrt{2}$

(D) $3+\sqrt{2}$