NARAYANA IIT ACADEMY - INDIA

IIT – JEE (2010) PAPER I QUESTION & SOLUTIONS (CODE 0)

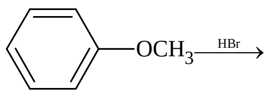
PART I : CHEMISTRY

PAPER – I

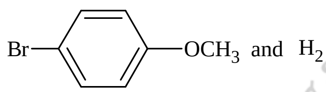
SECTION – I

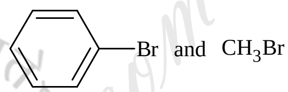
Single Correct Choice Type

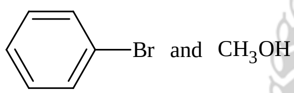
This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

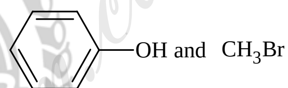
1. In the reaction 

the products are

(A) 

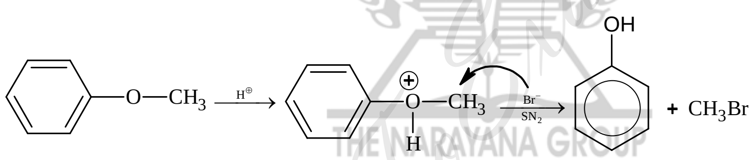
(B) 

(C) 

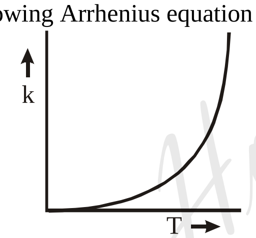
(D) 

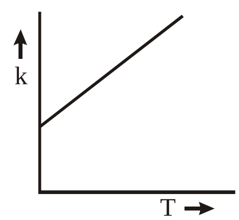
Key: **(D)**

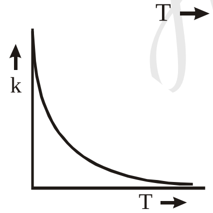
Sol.: $HBr \longrightarrow H^+ + Br^-$

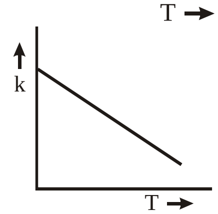


2. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that following Arrhenius equation is

(A) 

(B) 

(C) 

(D) 

Key: **(A)**

Sol.: $K = Ae^{\frac{-E_a}{RT}}$

Rate constant K increases exponentially with the rise in temperature. Since rate const. K also depends upon orientation factor A hence its maximum value is not at all infinity rather limited to an optimal value.

3. The species which by definition has ZERO standard molar enthalpy of formation at 298 K is

(A) Br_2 (g)
(B) Cl_2 (g)
(C) H_2O (g)
(D) CH_4(g)

Key: **(B)**

Sol.: Bromine and water exist in liquid state at 298 K. Methane is not an elemental species.

4. The ionization isomer of $\left[Cr(H_2O)_4Cl(NO_2)\right]Cl$ is

(A) $\left[Cr(H_2O)_4(O_2N)\right]Cl_2$ (B) $[Cr(H_2O)_4Cl_2](NO_2)$

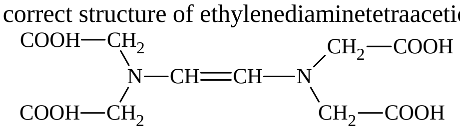
(C) $\left[Cr(H_2O)_4Cl(ONO)\right]Cl$ (D) $\left[Cr(H_2O)_4Cl_2(NO_2)\right]\cdot H_2O$

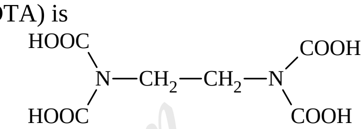
Key: **(B)**

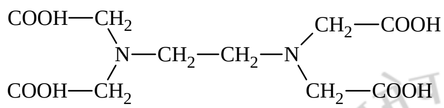
Sol.: $\left[Cr(H_2O)_4Cl(NO_2)\right]Cl \xrightleftharpoons{\text{ionization}} \left[Cr(H_2O)_4Cl(NO_2)\right]^+ + Cl^-$

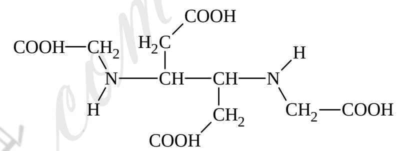
$\left[Cr(H_2O)_4Cl_2\right](NO_2) \xrightleftharpoons{\text{Ionization}} \left[Cr(H_2O)_4Cl_2\right]^+ + \bar{N}O_2$.

5. The correct structure of ethylenediaminetetraacetic acid (EDTA) is

(A) 

(B) 

(C) 

(D) 

Key: **(C)**

Sol.: Based on facts

6. The bond energy (in kcal mol^{-1}) of a C—C single bond is approximately

(A) 1 (B) 10

(C) 100 (D) 1000.

Key: **(C)**

Sol.: C – C single bond dissociation energy ranges between 88 to 150 K cal mol^{-1}.

7. The synthesis of 3–octyne is achieved by adding a bromolkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are

(A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$ (B) $BrCH_2CH_2CH_3$ and $CH_3CH_2CH_2C\equiv CH$

(C) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3C\equiv CH$ (D) $BrCH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$.

Key: **(D)**

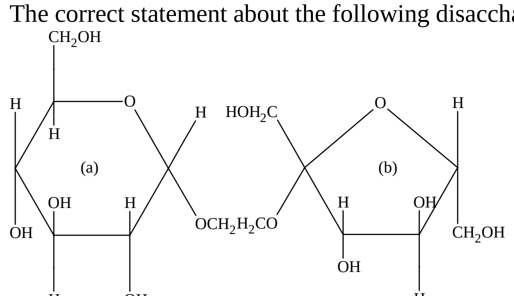
Sol.: $CH_3 - CH_2 - C \equiv C - C - CH_2 - CH_2 - CH_3$

3 – octyne

$$CH_3-CH_2-C\equiv CH \xrightarrow{NaNH_2} CH_3-CH_2-C\equiv \overset{\ominus}{C} \xrightarrow{CH_3-CH_2-CH_2-CH_2-Br} CH_3-CH_2-CH_2-CH_2-C\equiv C-CH_2-CH_3$$

3 octyne

8. The correct statement about the following disaccharide is



(A) Ring (a) is pyranose with α–glycosidic link
(B) Ring (a) is furanose with α–glycosidic link
(C) Ring (b) is furanose with α–glycosidic link
(D) Ring (b) is pyranose with β–glycosidic link.

Key: **(A)**

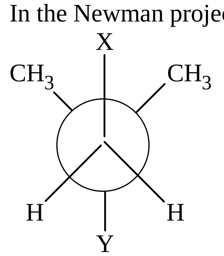
Sol.: Ring (a) is pyranose whereas ring(b) is furanose. α–anomeric form of ring (a) is attached through glycosidic bond.

SECTION – II

Multiple Correct Choice Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

9. In the Newman projection for 2, 2–dimethylbutane

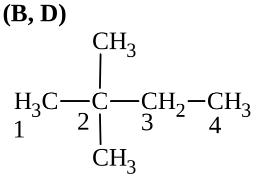


X and Y can respectively be

(A) H and H
(B) H and C_2H_5
(C) C_2H_5 and H
(D) CH_3 and CH_3.

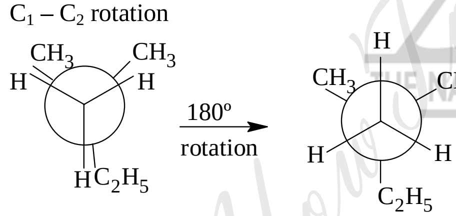
Key: **(B, D)**

Sol.:

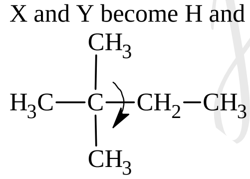


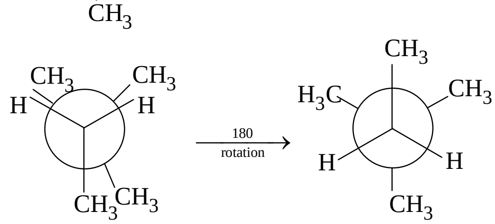
2, 2-dimethyl butane

$C_1 – C_2$ rotation



X and Y become H and C_2H_5





X and Y become CH_3 and CH_3.

10. The regent(s) used for softening the temporary hardness of water is (are)

(A) $Ca_3(PO_4)_2$
(B) $Ca(OH)_2$
(C) Na_2CO_3
(D) NaOCl.

Key: **(B, C)**

Sol.: Temporary hardness is due to the presence of bicarbonates of Ca and Mg. Temporary hardness can be removed by clarke's process which involves the addition of slaked lime

$$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3 + 2H_2O$$

Washing soda removes both the temporary and permanent hardness.

$$Ca(HCO_3)_2 + Na_2CO_3 \longrightarrow CaCO_3 + 2NaHCO_3.$$

11. Among the following, the intensive property is (properties are)

(A) molar conductivity
(B) electromotive force
(C) resistance
(D) heat capacity.

Key: **(A)**

Sol.: EMF, resistance and heat capacity are extensive properties ofcourse, resistivity is an intensive property.

12. Aqueous solutions of HNO_3, KOH, CH_3COOH, and CH_3COONa of identical concentrations are provided. The pair(s) of solutions which form a buffer upon mixing is(are)

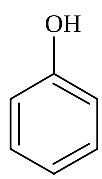
(A) HNO_3 and CH_3COOH
(B) KOH and CH_3COONa
(C) HNO_3 and CH_3COONa
(D) CH_3COOH and CH_3COONa .

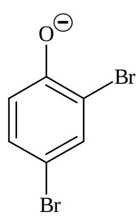
Key: **(C, D)**

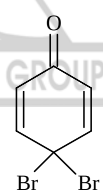
Sol.: Mixture of weak acid and its salt are known as acidic buffer.

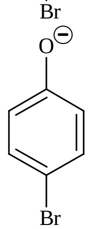
$$\underset{\text{strong acid}}{H^+NO_3^-} + CH_3CO_2^-Na^+ \longrightarrow \underset{\text{weak acid}}{CH_3CO_2H} + Na^+NO_3^-$$

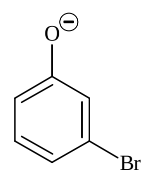
In an acid–based reaction. The equilibrium shifts to the direction which results in the formation of weaker acid.

13. In the reaction  $\xrightarrow{NaOH(aq)/Br_2}$ the intermediate(s) is (are)

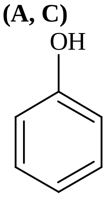
(A) 

(B) 

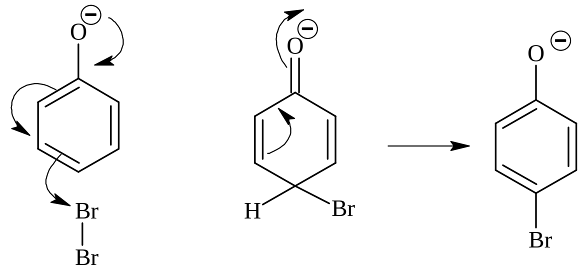
(C) 

(D) 

Key: **(A, C)**

Sol.: 

is strongly activating towards EAS reaction and it is ortho–para directing



SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon the first paragraph, 3 multiple choice questions and based upon the second paragraph 2 Multiple choice questions have to be answered. Each of these questions have four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 14 to 16

Copper is the most noble of the first row transition metals and occurs in small deposits in several countries. Ores of copper include chalcanthite ($CuSO_4$. $5H_2O$), atacamite ($Cu_2Cl(OH)_3$), cuprite (Cu_2O), copper glance (Cu_2S) and malachite ($Cu_2(OH)_2CO_3$). However 80% of the world copper production comes from the ore chalcopyrite ($CuFeS_2$). The extraction of copper from chalcopyrite involves partial roasting, removal of iron and self–reduction.

14. Partial roasting of chalcopyrite produces

(A) Cu_2S and FeO (B) Cu_2O and FeO

(C) CuS and Fe_2O_3 (D) Cu_2O and Fe_2O_3

Key: **(B)**

Sol: $CuFeS_2 + O_2 \rightarrow Cu_2S + 2FeS + SO_2$

The sulphites of copper and iron are partially oxidized

$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2$

$2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$.

15. Iron is removed from chalcopyrite as

(A) FeO (B) FeS

(C) Fe_2O_3 (D) $FeSiO_3$

Key: **(D)**

Sol: Fe is removed in the form of $FeSiO_3$.

$FeO + SiO_2 \longrightarrow FeSiO_3$

16. In self–reduction, the reducing species is

(A) S (B) O^{2-}

(C) S^{2-} (D) SO_2

Key: **(C)**

Sol: $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

S^{-2} oxidized into S^{+4} hence it is reducing species .

Paragraph for Questions 17 to 18

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is :

$$M(s) \,|\, M^+(aq;\ 0.05\ molar) \,||\, M^+(aq;\ 1 molar) \,|\, M(s)$$

For the above electrolytic cell the magnitude of the cell potential $|E_{cell}| = 70$ mV.

17. For the above cell

(A) $E_{cell} < 0;\ \Delta G > 0$ (B) $E_{cell} > 0;\ \Delta G < 0$

(C) $E_{cell} < 0;\ \Delta G^\circ > 0$ (D) $E_{cell} > 0;\ \Delta G^\circ < 0$

Key: **(B)**

Sol: $E_{cell} = \frac{-2.303RT}{F}\log\frac{0.05}{1}$ = a positive value

= 70 mV (given)

Hence $\Delta G < 0$.

18. If the 0.05 molar solution of M^+ is replaced by a 0.0025 molar M^+ solution, then the magnitude of the cell potential would be

(A) 35 mV (B) 70 mV

(C) 140 mV (D) 700 mV

Key: **(C)**

Sol: $E_{cell} = \frac{-2.303RT}{F}\log\frac{0.0025}{1}$

$= \frac{-2.303RT}{F}\log(0.05)^2$

= 2 × 70 mV

= 140 mV.

SECTION – IV
Integer Answer Type

This Section contains TEN questions. The answer to each question is a Single Digit Integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

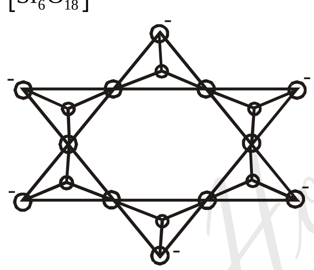
19. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is

Key: **(3)**

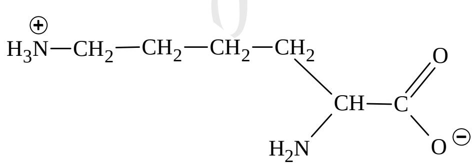
Sol.: $[Si_6O_{18}]^{12-}$

$Be_3Al_2Bi_6O_{18}$

$[Si_6O_{18}]^{12-}$

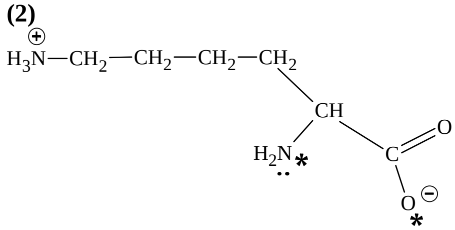


20. The total number of basic groups in the following form of lysine is



Key: **(2)**

Sol.:

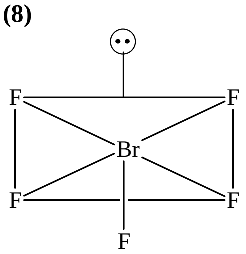


* Group are basic.

21. Based on VSEPR theory, the number of 90 degree F – Br – F angles in BrF_5 is

Key: **(8)**

Sol.:



The structure of BrF_5 is square pyramidal. The number of FBrF angles having the value of 90º is eight (8). Due to trivial distortion, however, the bond angles (F—Br—F) are slightly less than 90º(85º).

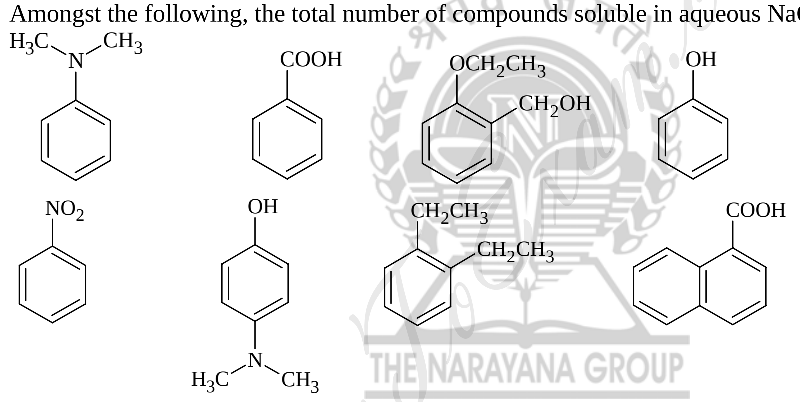
22. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is

KCN	K_2SO_4	$(NH_4)_2C_2O_4$	NaCl	$Zn(NO_3)_2$
$FeCl_3$	K_2CO_3	NH_4NO_3	LiCN	

Key: **(3)**

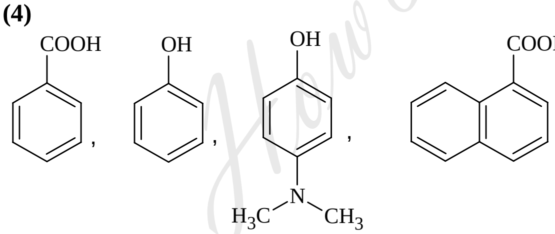
Sol.: KCN, K_2CO_3, LiCN are basic salt can convert red litmus to blue.

23. Amongst the following, the total number of compounds soluble in aqueous NaOH is



Key: **(4)**

Sol.:



are soluble in aq. NaOH.

24. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL, and 25.0 mL. The number of significant figures in the average titre value is

Key: **(3)**

Sol.: Average = $= \frac{25.2 + 25.25 + 25.0}{3}$

$= 75.45 / 3$

$= 25.15 \approx 25.1.$

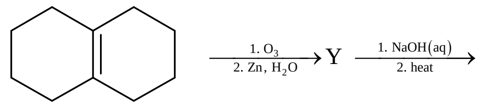
No. of significant figure = 3.

25. The number of neutrons emitted when $^{235}_{92}U$ undergoes controlled nuclear fission to $^{142}_{54}Xe$ and $^{90}_{38}Sr$ is

Key **(3)**

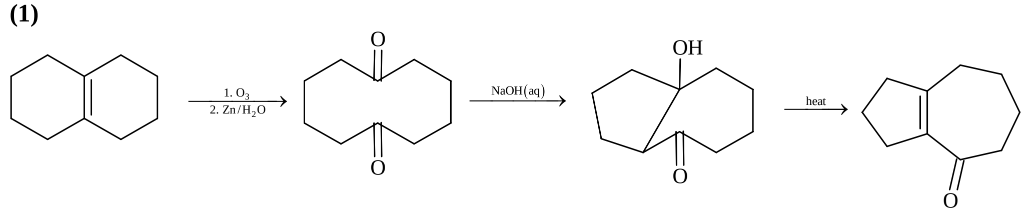
Sol.: $^{235}_{92}U \longrightarrow ^{142}_{54}Xe + ^{90}_{38}Sr + 3^{1}_{0}n$.

26. In the scheme given below, the total number of intramolecular aldol condensation products formed from 'Y' is



Key: **(1)**

Sol.:



27. The concentration of R in the reaction R → P was measured as a function of time and the following data is obtained :

[R] (moloar)	1.0	0.75	0.40	0.10
t (min.)	0.0	0.05	0.12	0.18

The order of the reaction is

Key: **(0)**

Sol.: $R \longrightarrow P$

$$-\frac{dc}{dt} \text{ after 0.05 min} = \frac{0.25}{0.05} = 5 \text{ M min}^{-1}$$

$$-\frac{dc}{dt} \text{ after 0.12 min} = \frac{060}{0.12} = 5 \text{ M min}^{-1}$$

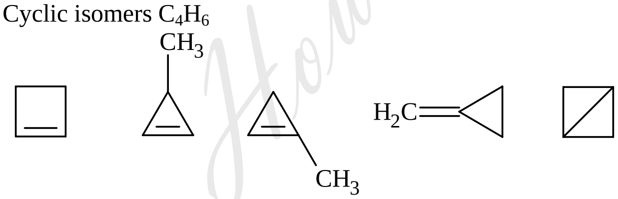
$$-\frac{dc}{dt} \text{ after 0.18 min} = \frac{90}{0.18} = 5 \text{ M min}^{-1}$$

The average rate remains same throughout. This implies that rate is independent of concentration (zero order).

28. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is

Key **(5)**

Sol.: Cyclic isomers C_4H_6



Total isomers = 5

PART II: MATHEMATICS

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

29. The number of 3 × 3 matrices A whose are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 (B) $2^9 - 1$

(C) 168 (D) 2

Key. (A)

Sol. Three planes cannot meet only at two distinct points. Hence 'A' is correct.

30. The value of $\lim\limits_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t\ln(1+t)}{t^4+4}\,dt$ is

(A) 0 (B) $\frac{1}{12}$

(C) $\frac{1}{24}$ (D) $\frac{1}{64}$

Key. (B)

Sol. $\lim\limits_{x\to 0} \frac{\int_0^x \frac{t\ln(1+t)\,dt}{t^4+4}}{x^3}$

$= \lim\limits_{x\to 0} \frac{x\ln(1+x)}{(x^4+4)3x^2} = \lim\limits_{x\to 0} \frac{\ln(1+x)}{3x(x^4+4)}$

$= \frac{1}{4\times 3}\lim\limits_{x\to 0}\frac{\ln(1+x)}{x} = \frac{1}{12}$.

31. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ (B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ (D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Key. (B)

Sol. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$\alpha^3 + \beta^3 = (\alpha + \beta)\{(\alpha + \beta)^2 - 3\alpha\beta\}$

$q = -p(p^2 - 3\alpha\beta)$

$\Rightarrow q + p^3 = 3\alpha\beta p \Rightarrow \alpha\beta = \frac{(q+p^3)}{3p}$

$\alpha^2 + \beta^2 = p^2 - 2\frac{(q+p^3)}{3p} = \frac{3p^3 - 2q - 2p^3}{3p} = \frac{p^3 - 2q}{3p}$

$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{p^3-2q}{q+p^3}$

$\Rightarrow x^2 - \frac{(p^3-2q)}{p^3+q}x + 1 = 0 \Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

32. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0
(C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

Key. (C)

Sol. $\vec{n}=(3\hat{i}+4\hat{j}+2\hat{k})\times(4\hat{i}+2\hat{j}+3\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 4 & 2\\ 4 & 2 & 3\end{vmatrix} = 8\hat{i}-\hat{j}-10\hat{k}$

The equation of plane containing the IInd and IIIrd given lines.

$\vec{r}.(8\hat{i}-\hat{j}-10\hat{k})=0 \Rightarrow 8x - y - 10z = 0.$

Now normal vector to the required plane is given by

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 8 & -1 & -10\end{vmatrix} = -26\hat{i}+52\hat{j}-26\hat{k}$$

$= -26(\hat{i}-2\hat{j}+\hat{k})$

The equation of the required plane is x – 2y + z = 0.

33. If the angle A, B and C of the triangle are in the an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$
(C) 1 (D) $\sqrt{3}$

Key. (D)

Sol. $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A = \frac{\sin A}{\sin C}2\sin C\cos C+\frac{\sin C}{\sin A}2\sin A\cos A$

$= 2\sin(A + C)$

$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

34. Let f, g and h be real–valued functions defined on the interval [0, 1] by $f(x) = e^{x^2}+e^{-x^2}$, $g(x) = xe^{-x^2}$ and $h(x) = x^2 + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(A) a = b and c ≠ b (B) a = c and a ≠ b
(C) a ≠ b (D) a = b = c

Key. (D)

Sol. $1 \ge x \ge x^2 \quad \forall x\in [0, 1]$

$e^{x^2} \ge xe^{x^2} \ge x^2e^{x^2} \quad \forall x\in [0, 1]$

i.e., $e^{-x^2}+e^{x^2} \ge e^{-x^2}+xe^{x^2} \ge e^{-x^2}+x^2e^{x^2}$

equality holds when x = 1

i.e., $f(x) \ge g(x) \ge h(x)\ \forall x\in [0, 1]$

Hence a = b = c.

35. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$

(C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Key. (C)

Sol. Required prob. = $\frac{2\times2\times2(3!)}{6\times6\times6} = \frac{2}{9} \Rightarrow \frac{2}{9}$

36. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square (C) rectangle, but not a square

(D) rhombus, but not a square

Key. (A)

Sol. $P: -2\hat{i}-\hat{j},\ Q: 4\hat{i},\ R: 3\hat{i}+3\hat{j},\ S: -3\hat{i}+2\hat{j}$

$\overrightarrow{PQ} =$ of P $= 6\hat{i}+\hat{j}$

$\overrightarrow{QR} = 3\hat{i}+3\hat{j}-4\hat{i} = -\hat{i}+3\hat{j}$

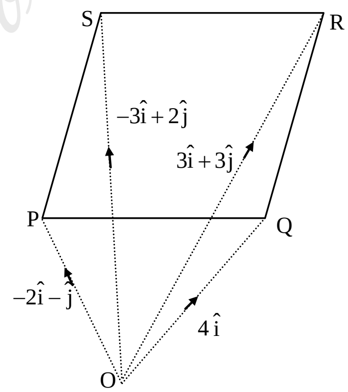
$\overrightarrow{PS} = -3\hat{i}+2\hat{j}+2\hat{i}+\hat{j} = -\hat{i}+3\hat{j}$

$\overrightarrow{SR} = 3\hat{i}+3\hat{j}+3\hat{i}-2\hat{j} = 6\hat{i}+\hat{j}$

$\overrightarrow{PQ}.\overrightarrow{PS} = (6\hat{i}+\hat{j}).(-\hat{i}+3\hat{j}) = -3 \neq 0$

Here $\overrightarrow{PQ} \parallel \overrightarrow{SR}$ and $\overrightarrow{PS} \parallel \overrightarrow{QR}$

but $\overrightarrow{PQ}$ is not perpendicular to $\overrightarrow{PS}$



SECTION – II

Multiple Correct Choice Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

37. Let z_1 and z_2 be two distinct complex numbers and let $z = (1-t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

(A) $|z-z_1| + |z-z_2| = |z_1-z_2|$ (B) $\text{Arg}(z-z_1) = \text{Arg}(z-z_2)$

(C) $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix} = 0$ (D) $\text{Arg}(z-z_1) = \text{Arg}(z_2-z_1)$

Key. (A, C, D)

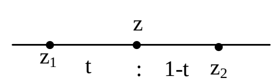
Sol. As $z = (1-t)z_1 + tz_2$

$\Rightarrow z_1, z, z_2$ are collinear

$\therefore$ A, D are correct

Also $\frac{z-z_1}{z_2-z_1} = \frac{\bar{z}-\bar{z}_1}{\bar{z}_2-\bar{z}_1}$

$\therefore$ (C) is correct.



38. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$

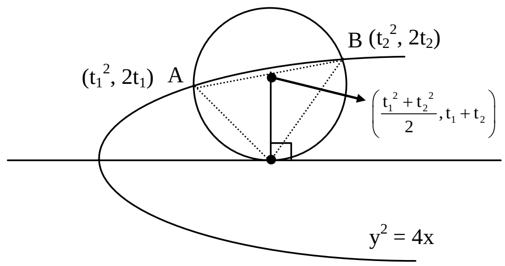
(C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Key, (C, D)

Sol. Slope of line AB

$M = \frac{(t_2 - t_1)}{(t_2 - t_1)(t_2 + t_1)} = \left(\frac{2}{t_1 + t_2}\right) = \pm\frac{2}{r}$

As $|t_1 + t_2| = r$



39. Let f be a real–valued function defined on the interval $(0, \infty)$ by $f(x) = \ln x + \int_0^x \sqrt{1 + \sin t}\, dt$. Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x \in (0, \infty)$

(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$

(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$

(D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$

Key. (B, C)

Sol. $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}\, dt, \; x > 0$

$f'(x) = \frac{1}{x} + \sqrt{1 + \sin x}, \; x > 0$

Clearly $f'(x)$ exists for all $x \in (0, \infty)$

and $f'(x)$ is continuous on $(0,\infty)$

but not differentiable on $(0,\infty)$

More over $f'(x), f(x) > 0 \; \forall x \in (1, \infty)$

and $\ln x + \int_0^x \sqrt{1+\sin t}\, dt > \frac{1}{x} + \sqrt{1+\sin x} \; \forall x \in (\pi, \infty)$

$\frac{1}{x}$ is not bounded.

$\therefore$ (D) is incorrect .

Hence, option B, C are correct.

40. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is (are)

(A) $\frac{22}{7} - \pi$ (B) $\frac{2}{105}$

(C) 0 (D) $\frac{71}{15} - \frac{3\pi}{2}$

Key. (A)

Sol. $\int_0^1 \frac{x^4(1-x)^4}{(1+x^2)} dx$

$\int_0^1 (x^6 - 4x^5 + 5x^4) dx - \int_0^1 \frac{4x^4}{1+x^2} dx = \frac{10}{21} - 4\int_0^1 \frac{x^4 - 1 + 1}{x^2 + 1} dx = \frac{22}{7} - \pi$

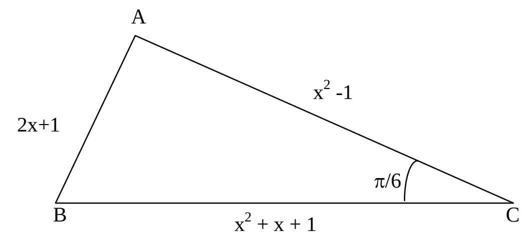
41. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

Key. (B)

Sol.



$$\frac{\sqrt{3}}{2} = \frac{(x^2+x+1)^2+(x^2-1)^2-(2x+1)^2}{2.(x^2-1)(x^2+x+1)}$$

$$\frac{\sqrt{3}}{2} = \frac{(x^2+3x+2)(x^2-x)+(x^2-1)^2}{2(x^2-1)(x^2+x+1)}$$

$$\frac{\sqrt{3}}{2} = \frac{(x+2)x+x^2-1}{2(x^2+x+1)}$$

$$\sqrt{3} = \frac{2x^2+2x-1}{x^2+x+1}$$

$$\Rightarrow x^2\left(\sqrt{3}-2\right)+x\left(\sqrt{3}-2\right)+\sqrt{3}+1=0$$

$$x = \frac{-(\sqrt{3}-2)\pm\sqrt{(\sqrt{3}-2)^2-4(\sqrt{3}-2)(\sqrt{3}+1)}}{2(\sqrt{3}-2)} = \sqrt{3}+1.$$

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon the first paragraph, 3 multiple choice questions and based upon the second paragraph 2 Multiple choice questions have to be answered. Each of these questions have four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 42 to 44

Let p be an odd prime number and T_p be the following set of 2 × 2 matrices.

$$T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, ..., p-1\}\right\}$$

Sol. 42 to 44

as A is symmetric b = c

det A = $a^2 - b^2 = (a + b)(a - b)$

a, b, c, ∈ {0, 1, 2, ….. p – 1}

no. of numbers of type

np = 1

np + 1 = 1

np + 2 = 1 $n \in I$

⋮

np + (p – 1) = 1

42. The number of A in T_p such that A is either symmetric or skew–symmetric or both, and det(A) divisible by p is

(A) $(p-1)^2$ (B) $2(p-1)$

(C) $(p-1)^2+1$ (D) $2p-1$

Key. (D)

Sol. as det(A) is div. by p ⇒ either a + b div. by p corresponding nu. Of ways = (p – 1) [excluding zero]

or (a – b) is div. by p corresponding no. of ways = p

Total number of ways = 2p – 1

43. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is
[Note: The trace of a matrix is the sum of its diagonal entries.]

(A) $(p - q)(p^2 p p + q)$ (B) $p^3 - (p-1)^2$
(C) $(p-1)^2$ (D) $(p-1)(p^2 - 2)$

Key. (C)

Sol. as Tr(A) not div. by p $\Rightarrow a \neq 0$
det(A) is div. by p $\Rightarrow a^2 - bc$ div. by p
no. of ways of selection of a, b, c
$$(p-1)[(p-1) \times 1] = (p-1)^2$$

44. The number of A in T_p such that det (A) is not divisible by p is

(A) $2p^2$ (B) $p^3 - 5p$
(C) $p^3 - 3p$ (D) $p^3 - p^2$

Key. (D)

Sol. Total number of A = $p \times p \times p = p^3$
No. of A such that det(A) div. by p
$= (p-1)^2$ + no. of A in which a = 0
$= (p-1)^2 + p + p - 1$
$= p^2$
required no. = $p^3 - p^2$.

Paragraph for Questions Nos. 45 to 46

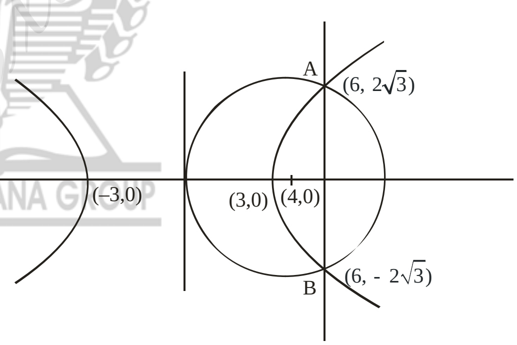
The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

45. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$
(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Key. (B)

Sol. Equation of tangent at point P(θ)
$$\frac{x\sec\theta}{3} - \frac{y\tan\theta}{2} - 1 = 0 \quad \text{.....(i)}$$
since eq. (i) will be a tangent to the circle
$$\therefore \frac{\frac{4\sec\theta}{3} - 1}{\sqrt{\frac{\sec^2\theta}{9} + \frac{\tan^2\theta}{4}}} = 4$$
by solving it we will get
$$2x - \sqrt{5}y + 4 = 0$$

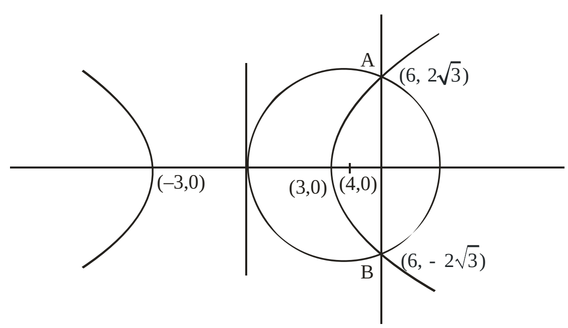


46. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$ (B) $x^2 + y^2 + 12x + 24 = 0$
(C) $x^2 + y^2 + 24x - 12 = 0$ (D) $x^2 + y^2 - 24x - 12 = 0$

Key. (A)

Sol. $\frac{x^9}{9} = 1 + \frac{(-x^2 + 8x)}{4}$
$4x^2 = 36 + 9(-x^2 + 8x)$
$13x^2 - 72x - 36 = 0$
$x = 6,$
$y = \pm 2\sqrt{3}$
Required equation of circle is
$(x-6)^2 + y^2 - 12 = 0$
$x^2 + y^2 - 12x + 24 = 0$



SECTION – IV
Integer Answer Type

This Section contains TEN questions. The answer to each question is a Single Digit Integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

47. Let ω be the complex number $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \text{ is equal to}$$

Key. (1)

Sol. $\omega = \cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3} = -\frac{1}{2}+i\frac{\sqrt{3}}{2}$

ω is one of cube root of unity.

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \qquad \left[\because 1+\omega+\omega^2 = 0\right]$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$ gives

$$\begin{vmatrix} 0 & 0 & z \\ \omega-z-\omega^2 & z+\omega^2-1 & 1 \\ \omega^2-1 & 1-z-\omega & z+\omega \end{vmatrix} = 0$$

$$z\left[(\omega-z-\omega^2)(1-z-\omega)-(\omega^2-1)(z+\omega^2-1)\right] = 0$$

$z[z^2] = 0$

$\Rightarrow \quad z^3 = 0$

$= z = 0$

Ans. is $= 1$

48. The number of values of θ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ± 1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

Key. (3)

Sol. $\tan\theta = \cot 5\theta$

$\tan\theta = \tan\left(\frac{\pi}{2}-5\theta\right)$

$\theta = n\pi+\frac{\pi}{2}0-5\theta$

$6\theta = n\pi+\frac{\pi}{2}$

$\theta = \frac{n\pi}{6}+\frac{\pi}{12} n\in I \quad .....(i)$

$\sin 2\theta = \cos 4\theta$

$\sin 2\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow \quad 2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$\Rightarrow \quad 2\sin^2 2\theta + 2\sin 2\theta - \sin 2\theta - 1 = 0$

$(2\sin 2\theta - 1)(\sin 2\theta + 1) = 0$

$\sin 2\theta = \frac{1}{2}, \qquad \sin 2\theta = -1$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \qquad 2\theta = -\frac{\pi}{6}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12} \qquad \theta = -\frac{\pi}{4}$

All three values of θ which satisfy the eq. (i).

49. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$. Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x \, dx$ is

Key. (4)

Sol. $f(x) = \begin{cases} 1 - \{x\} & , \ 0 \le x < 1 \\ \{x\} & , \ 1 \le x < 2 \\ 1 - \{x\} & , \ 2 \le x < 3 \end{cases}$

Here f(x) is periodic with period 2 and cos πx is also periodic with period 2

$\therefore$ f(x) cos πx is periodic with period "2".

$\int_{-10}^{10} f(x)\cos \pi x \, dx = 10\int_{0}^{2} f(x)\cos \pi x \, dx = \frac{40}{\pi^2}$

Hence, $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x \, dx = \frac{\pi^2}{10} \times \frac{40}{\pi^2} = 4.$

50. If the distance between the plane Ax – 2y + z = d and the plane containing the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |d| is

Key. (6)

Sol. The equation of the plane containing the given lines will be a(x – 1) + b(y – 2) + c(z – 3) = 0 where a, b, c are direction ratios of normal to the plane considering vectors parallel to the two lines

2i + 3j + 4k and 3i + 4j + 5k

So $2a_1 + 3b_1 + 4c_1 = 0$

$3a_1 + 4b_1 + 5c_1 = 0$

$\frac{a_1}{15-16} = \frac{-b_1}{10-12} = \frac{c_1}{8-9}$

So the plane is x – 2y + z = 0

Hence distance between two planes

$\frac{|d|}{\sqrt{1^2 + 2^2 + 1}} = \sqrt{6}$

$|d| = 6$

51. The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x–axis, then the eccentricity of the hyperbola is

Key. (2)

Sol. Since the line 2x + y – 1 = 0 is tangent

so, $C^2 = a^2m^2 - b^2$

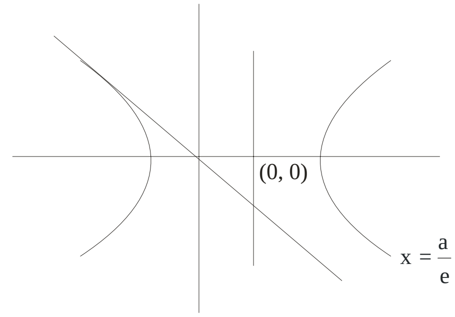
$1 = 4a^2 - b^2$ …..(i)

Also line passes through $\left(-\frac{a}{e}, 0\right)$

So, $2\left(-\frac{a}{e}\right)=1$

$4a^2 = e^2$ ……(ii)

Using (i) and (ii) e = 2



52. Let S_k, k = 1, 2, ... , 100, denote the sum of the infinite geometric series whose whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right|$ is

Key. (4)

Sol. $S_k = \frac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$

We have $S_1 = 1$

$S_2 = 1$

$S_3 = \frac{1}{2}$

Now, $\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right|$

$= S_1 + S_2 + S_3 + \sum_{k=4}^{100}\frac{\left(k^2-3k+1\right)}{(k-1)!}$

$= 1 + 1 + \frac{1}{2}+\sum_{k=4}^{100}\left[\frac{1}{(k-3)!}-\frac{1}{(k-1)!}\right]$

$= 1 + 1 + \frac{1}{2}+\left[1+\frac{1}{2!}-\frac{1}{98!}-\frac{1}{99!}\right]$

$= 4 - \frac{100}{99!}$

So, $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right| = 4$.

53. Let f be a real–valued differentiable function on R (the set of all real numbers) such that f(1) = 1. If the y–intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to

Key. (9)

Sol. eq. of tangent at P(x, y)

$$Y - y = \frac{dy}{dx}(X - x)$$

y–integer $y - x\frac{dy}{dx} = x^3$

$$\frac{dy}{dx} - \frac{y}{x} = -x^2$$

I.F. = $e^{-\int \frac{1}{x}dx} = \frac{1}{x}$

The solution

$$y \times \frac{1}{x} = \int -x^2 \times \frac{1}{x}dx$$

$$\frac{y}{x} = -\frac{x^2}{2} + C$$

$$f(1) = 1 \quad \Rightarrow \quad C = \frac{3}{2}$$

$$f(x) = y = \frac{3x - x^3}{2}$$

$$f(-3) = 9$$

54. If $\bar{a}$ and $\bar{b}$ are vectors in space given by $\bar{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\bar{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$ is

Key. (5)

Sol. $|\bar{a}| = |\bar{b}| = 1 \qquad \bar{a}.\bar{b} = 0$

Let $\vec{l} = (\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b}) = (\vec{a} \times \vec{b}) \times \vec{a} - 2(\vec{a} \times \vec{b}) \times \vec{b}$

$= |a|^2\ \vec{b} - (\vec{a}.\vec{b})\vec{a} - 2(a.b)\vec{b} + 2|b|a.$

$= \vec{b} + 2\vec{a}$

$(2\vec{a} + \vec{b}).\vec{l} = |2\vec{a} + \vec{b}|^2 = 5$

55. The number of all possible values of θ, where 0 < θ < π, for which the system of equations

(y + z) cos3θ = (xyz) sin 3θ

xsin3θ = $\frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

(xyz) sin3θ = (y + 2z) cos3θ + ysin 3θ have a solution (x_0, y_0, z_0) with $y_0\, z_0 \neq 0$, is

Key. (3)

Sol. $(y + z)\cos 3\theta = (xyz)\sin 3\theta$ (A)

$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$ (B)

$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$ (C)

When cos 3θ ≠ 0.

$\tan 3\theta = \frac{y+z}{xyz} = \frac{2z}{y(xz-2)} = \frac{y+2z}{xyz-y}$

as $y \neq 0$

$(y + z)(xz - 2) = 2z(xz)$

$xyz + xz^2 - 2z - 2y = 2xz^2$

$xyz = 2y + 2z + xz^2$ …..(i)

Again, $2z(xz - 1) = (y + 2z)(xz - 2)$

$2xz^2 - 2z = xyz + (2xz^2 - 4z - 2y)$

$xyz = 2y + 2z$ …..(ii)

from (i) and (ii) $xz^2 = 0$

$\Rightarrow$ $x = 0$ as $z \neq 0$

from (A) $(y + z)\cos 3\theta = 0$

$\Rightarrow$ $y + z = 0$

But when $\cos 3\theta = 0$ from (B)

$\sin 3\theta = 0$ not possible

So $y = -z$ putting in (B) and (C)

$x = 0$

$\sin 3\theta = \cos 3\theta$

$\Rightarrow$ $\tan 3\theta = 1 \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

56. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Key. (2)

Sol. Let $y = \frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta} = \frac{1}{3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta}$

$-\frac{5}{2} \leq 2\cos 2\theta + \frac{3}{2}\sin 2\theta \leq \frac{5}{2}$

max. value of $y = \frac{1}{3 - \frac{5}{2}} = 2$

PART III: PHYSICS

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

57. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(A) the bulb glows dimmer
(B) the bulb glows brighter
(C) total impedance of the circuit is unchanged
(D) total impedance of the circuit increases.

Key. (B)

Sol. $P = V_{rms} I_{rms} \cos\phi$

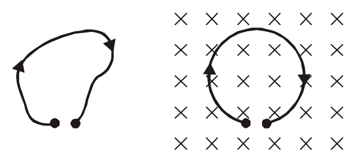
$= \frac{V_{rms}^2}{Z} \cdot \frac{R}{Z}$

$= \frac{V_{rms}^2 R}{Z^2}$

$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

As ω increase Z, decreases, so P increases.
Hence correct option is (B).

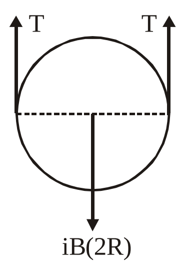
58. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(A) IBL
(B) $\frac{IBL}{\pi}$
(C) $\frac{IBL}{2\pi}$
(D) $\frac{IBL}{4\pi}$.

Key. (C)

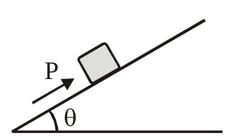
Sol. $2T = iB(2R)$



$T = \frac{iBL}{2\pi}$

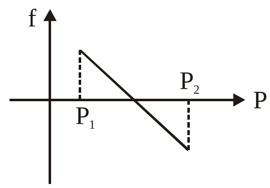
Hence correct option is (C).

59. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2\ mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph

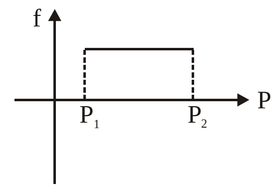


will look like

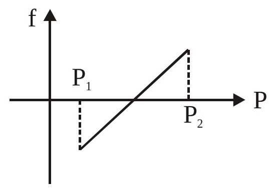
(A)



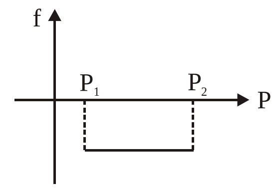
(B)



(C)



(D)



Key. (A)

Sol. $P_1 = mg(\sin\theta - \mu\cos\theta)$

$\text{friction}_{\text{initial}} = \mu mg\cos\theta$ up along the plane

$\text{friction}_{\text{final}} = \mu mg\cos\theta$ down along the plane

Hence correct option is (A).

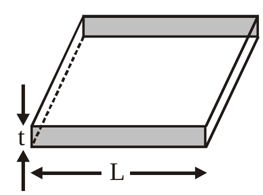
60. A real gas behaves like an ideal gas if its

(A) pressure and temperature are both high
(B) pressure and temperature are both low
(C) pressure is high and temperature is low
(D) pressure is low and temperature is high.

Key. (D)

Sol. For ideal gas behaviour pressure should be low and temperature should be high.
Hence correct option is (D).

61. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is



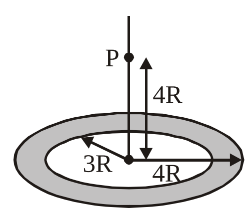
(A) directly proportional to L
(B) directly proportional to t
(C) independent of L
(D) independent of t.

Key. (C)

Sol. $R = \rho\frac{\ell}{A} = \rho\cdot\frac{L}{Lt} = \frac{\rho}{t}$

Hence correct option is (C).

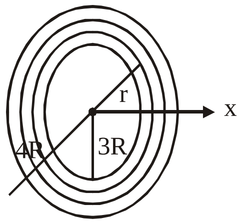
62. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is



(A) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$
(B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$
(C) $\frac{GM}{4R}$
(D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$.

Key. (A)

Sol.



$$dV = -\frac{G\cdot\sigma\cdot 2\pi r dr}{\sqrt{r^2+x^2}}$$

$$V = -2\pi G\sigma\int_{3R}^{4R}\frac{rdr}{\sqrt{r^2+x^2}}$$

$$r^2+x^2 = z$$

$$2r\,dr = dz$$

$$\int\frac{rdr}{\sqrt{r^2+x^2}} = \frac{dz}{2\sqrt{z}}$$

$$= \frac{1}{2}\frac{z}{1/2} = \sqrt{z}$$

$$V = -2\pi G\sigma\left[\sqrt{r^2+x^2}\right]_{3R}^{4R}$$

$$= -2\pi Gr\left[4R\sqrt{2}-5\right]$$

$$W = (1)\left[0+2\pi G\sigma(4R\sqrt{2}-5R)\right]$$

$$= 2\pi G\cdot\frac{M}{\pi(16-9)R^2}(4R\sqrt{2}-5R)$$

$$= \frac{2\pi GM}{7R}(4\sqrt{2}-5)\ .$$

Hence correct option is (A).

63. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$

(B) $R_{100} = R_{40} + R_{60}$

(C) $R_{100} > R_{60} > R_{40}$

(D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$.

Key. (D)

Sol. $R = \frac{v^2}{P}$

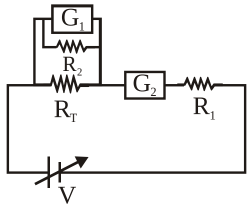
As temperature increase, resistance increases

So, $R_{40} > R_{60} > R_{100}$.

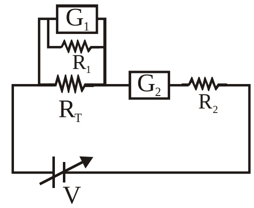
Hence correct option is (D).

64. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

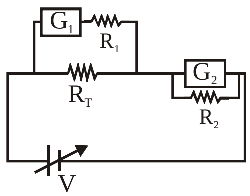
(A)



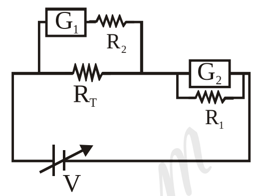
(B)



(C)



(D)



Key. (C)

Sol. An ideal voltmeter should have large resistance and an ideal ammeter should have low resistance.
Hence correct option is (C).

SECTION – II

Multiple Correct Choice Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

65. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ?

(A) total momentum of the system is 3 kg ms^{-1}

(B) momentum of 5 kg mass after collision is 4 kg ms^{-1}

(C) kinetic energy of the center of mass is 0.75 J

(D) total kinetic energy of the system is 4 J.

Key. (A), (C)

Sol.

$(1)(V) + (5)(0) = (1)(-2) + 5V'$

$V = 5V' - 2$ …(i)

$$\frac{V'+2}{V-0} = 1$$

$V' = V - 2$ …(ii)

$V = 5(V-2) - 2$

From equation (i) and (ii)

$V = 5V - 10 - 2$

$4V = 12$

$V = 3$ m/s.

$P_i = (1)(3) = 3$ kg – m/s.

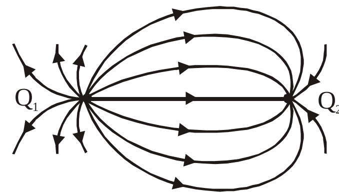
$$V_{CM} = \frac{(1)(3)+(5)(0)}{6} = \frac{1}{2}\,\text{m/s}$$

$$K_{CM} = \frac{1}{2}(6)\frac{1}{4} = \frac{3}{4} = 0.75\ \text{J}$$

$$K_{total} = \frac{1}{2}(1)(3)^2 = 4.5\ \text{J}$$

Hence correct options are (A), (C).

66. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x–axis are shown in the figure. These lines suggest that



(A) $|Q_1| > |Q_2|$
(B) $|Q_1| < |Q_2|$
(C) at a finite distance to the left of Q_1 the electric field is zero
(D) at a finite distance to the right of Q_2 the electric field is zero.

Key. (A), (D)

Sol. Density of field lines is more are Q_1

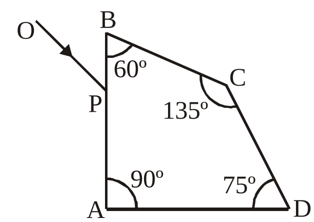
$\therefore$ $|Q_1| > |Q_2|$

Q_1 and Q_2 are of opposite signs

So, null point will be closer to charge of smaller magnitude i.e., Q_2

Hence correct options are (A), (D).

67. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60º (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct ?



(A) the ray gets totally internally reflected at face CD
(B) the ray comes out through face AD
(C) the angle between the incident ray and the emergent ray is 90º
(D) the angle between the incident ray and the emergent ray is 120º.

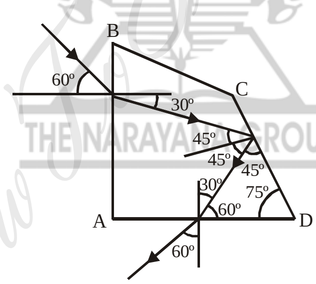
Key. (A), (B), (C)

Sol. $1\sin 60 = \sqrt{3}\sin r$

$r = 30^\circ$

$\sin\theta_C = \dfrac{1}{\sqrt{3}}$

$\theta_C \simeq 35^\circ$



At CD angle of incidence is greater than θ_C.

At AD angle of incidence is less than critical angle

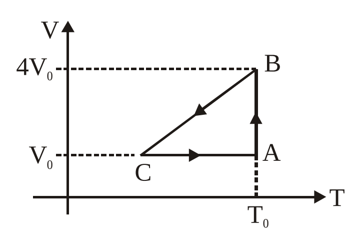
So ray will come out of AD.

Angle of deviation

$-30 + 90 + 30 = 90^\circ$

Hence correct options are (A), (B), (C)

68. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in figure. Its pressure at A is P_0. Choose the correct option(s) from the following :



(A) internal energies at A and B are the same
(B) work done by the gas is process AB is $P_0V_0\ell n 4$
(C) pressure at C is $\dfrac{P_0}{4}$
(D) temperature at C is $\dfrac{T_0}{4}$.

Key. (A), (B)

Sol. Internal energy of an ideal gas depends on temperature

$$W_{BC} = nRT\,\ell n\frac{V_2}{V_1}$$

$$= (1)(R)\frac{P_0V_0}{R}\ell n\frac{4V_0}{V_0}$$

$$= P_0V_0\ell n4$$

Hence (A), (B) options are correct.

69. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 second fore this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true ?

(A) error ΔT in measuring T, the time period, is 0.05 seconds
(B) error ΔT in measuring T, the time period, is 1 second
(C) percentage error in the determination of g is 5%
(D) percentage error in the determination of g is 2.5%.

Key. (A), (C)

Sol. Error in measurement of $T = \frac{1}{20}s = 0.05$ s

$$\frac{dg}{g} = 2\frac{dT}{T}$$

$$\frac{dg}{g} = 2\times\frac{1}{40}$$

% error in calculation of g = 5%.

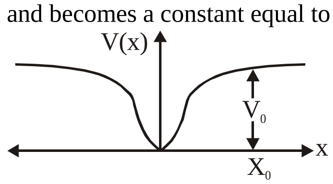
SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs. Based upon the first paragraph, 3 multiple choice questions and based upon the second paragraph 2 Multiple choice questions have to be answered. Each of these questions have four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 70 to 72

When a particle of mass m moves on the x–axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x–axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



70. If the total energy of the particle is E, it will perform periodic motion only if

(A) $E < 0$
(B) $E > 0$
(C) $V_0 > E > 0$
(D) $E > V_0$.

Key. (C)

Sol. For periodic motion
Total energy should be less than V_0 but greater than zero.
Hence (C) is correct.

71. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$.

Key. (B)

Sol. Dimensionally only B is correct.

72. The acceleration of this particle for $|x| > X_0$ is

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$

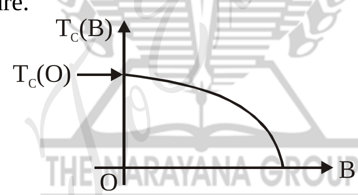
(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) zero.

Key. (D)

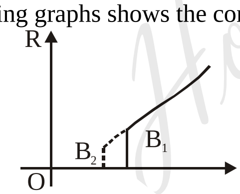
Sol. For $x > x_0$
potential energy is constant
force on particle is zero.
Hence (D) is correct.

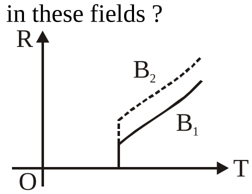
Paragraph for Question Nos. 73 to 74

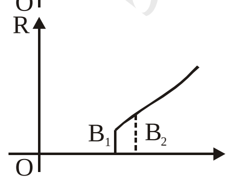
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure.

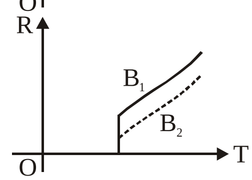


73. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic field B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields ?

(A)


(B)


(C)


(D)


Key. (A)

Sol. As B increases, critical temperature decreases.

74. A superconductors has $T_C(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

(A) B = 5 Tesla, $T_C(B) = 80$ K (B) B = 5 Tesla, 75 K < $T_C(B)$ < 100 K

(C) B = 10 Tesla, 75 K < $T_C(B)$ < 100 K (D) B = 10 Tesla, $T_C(B) = 70$ K.

Key. (B)

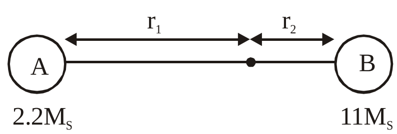
SECTION – IV
Integer Answer Type

This Section contains TEN questions. The answer to each question is a Single Digit Integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

75. A binary star consists of two starts A (mass 2.2. M_S) and B (mass 1 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their center of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the center of mass is.

Key. 6.

Sol.



$$\frac{11r_2^2+2.2r_1^2}{11r_2^2}$$

$$11\,r_2 = 2.2\,r_1$$

$$=1+\frac{2.2}{11}\cdot\frac{r_1^2}{r_2^2}$$

$$=1+\frac{2.2}{11}\times\left(\frac{11}{2.2}\right)^2=6\,.$$

76. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

Key. 6.

Sol. $$m=\frac{|f|}{|f-u|}$$

$$\frac{m_{25}}{m_{50}}=6\,.$$

77. A 0.1 kg mass is suspended form a wire of negligible mass. The length of the wire is 1 m and its cross–sectional area is $4.9\times10^{-7}\,m^2$. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n\times10^9\,Nm^{-2}$, the value of n is

Key. 4.

Sol. $$\omega^2=\frac{K}{m}$$

$$140\times140=\frac{YA}{\ell m}=\frac{Y(4.9\times10^{-7})}{(1)(0.1)}$$

$$140\times140=y(49)\times10^{-7}$$

$$y=4\times10^9$$

n = 4.

78. When two progressive waves y_1 4 sin (2x – 6t) and y_2 = 3 $\sin\left(2x-6t-\frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

Key. 5.

Sol. Amplitude $= \sqrt{4^2 + 3^2} = 5$.

79. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of total energy radiated by A to that of B ?

Key. 9.

Sol. (T_1) (500 nm) = T_2 (1500 nm)

$T_1 = 3T_2$

$E_A = \sigma \cdot 4\pi(6cm)^2 (T_1)^4$

$E_B = \sigma \cdot 4\pi(18cm)^2 (T_1)^4$

$$\frac{E_A}{E_B} = \left(\frac{1}{3}\right)^2 \times (3)^4 = 9.$$

80. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be

Key. 3.

Sol.
$$\frac{GM_P}{R_P^2} = \frac{\sqrt{6}}{11} g = \frac{\sqrt{6}}{11}\frac{GM_e}{R_e^2} \quad \ldots(i)$$

$\sqrt{2g_eR_e} = 11 \text{ km/s}$

$\sqrt{2g_PR_P} = x$

$$\frac{g_eR_e}{g_PR_P} = \frac{(11)^2}{x^2}$$

$$\frac{\frac{GM_e}{R_e^2}\cdot R_e}{\frac{GM_P}{R_P^2}\cdot R_P} = \frac{121}{x^2}$$

$$\frac{\frac{M_e}{R_e}}{\frac{M_P}{R_P}} = \frac{121}{x^2} \quad \ldots(ii)$$

$$\frac{M_P}{R_P^3} = \frac{2}{3}\cdot\frac{M_e}{R_e^3} \quad \ldots(iii)$$

x = 3.

81. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

Key. 7.

Sol.
$$f_1 = \frac{V + V_{C_1}}{V - V_{C_1}} f_0 ; \; f_2 = \frac{V + V_{C_2}}{V - V_{C_2}} f_0$$

$$\Delta f = \left[\frac{V + V_{C_1}}{V - V_{C_1}} - \frac{V + V_{C_2}}{V - V_{C_2}}\right] f_0 = \frac{1.2}{100} f_0$$

$$= \frac{2\Delta V_C}{V} f_0 = \frac{1.2 f_0}{100}$$
$$\Delta V_C = 7 \text{ km/hr}.$$

82. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ then the value of R in Ω is

Key. 4.

Sol.
$$J_1 = \left(\frac{2\varepsilon}{2+R}\right)^2 R$$
$$J_2 = \left(\frac{\varepsilon}{0.5+R}\right)^2 R$$
$$2.25 = \frac{4(0.5+R)^2}{(2+R)^2}$$
$$\frac{9}{4} = \frac{4(R+0.5)}{2+R}$$
$$\frac{3}{2} = \frac{2R+2}{2+R}$$
$$6+3R = 4R+2$$
$$R = 4\Omega.$$

83. A piece of ice (heat capacity = 2100 J kg^{-1} ^{o}C^{-1} and latent heat = 3.36 × 10^5 J kg^{-1}) of mass m grams is at –5ºC at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice–water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Key. 8.

Sol.
$$\left[m(2100)(5)+1(3.36\times10^5)\right]\times10^{-3} = 420$$
$$11m+336 = 420$$
$$11m = 420-336 = 84$$
$$m = 8 \text{ gm}.$$

84. An α–particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

Key. 3.

Sol.
$$\lambda = \frac{h}{\sqrt{2mk}}$$
$$k = qV$$
$$\lambda = \frac{h}{\sqrt{2(m)qV}}$$
$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\lambda}{m_p q_p}} = \sqrt{\frac{(4m)(2q)}{(m)q}} = 2\sqrt{2} = 3.$$

IIT – JEE (2010) PAPER II QUESTION & SOLUTIONS CODE 0

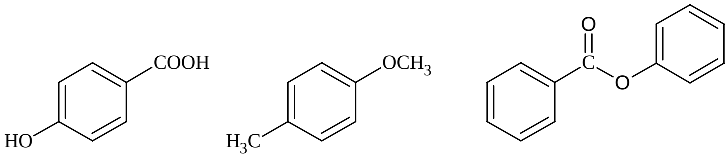
PART I : CHEMISTRY

PAPER – II

SECTION – I
Single Correct Choice Type

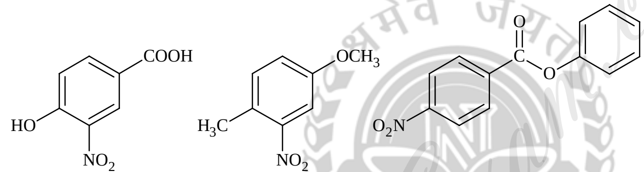
This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

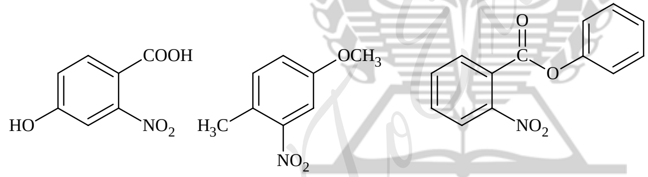
1. The compounds P, Q and S

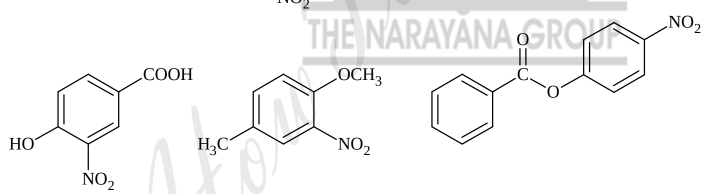


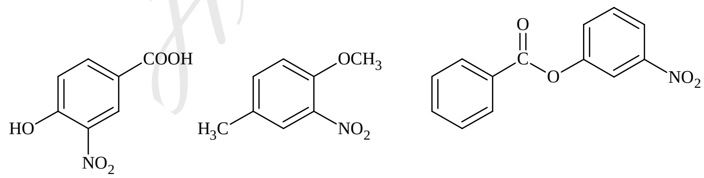
P **Q** **S**

where separately subjected to nitration using HNO_3/H_2SO_4 mixture. The major product formed in each case respectively, is

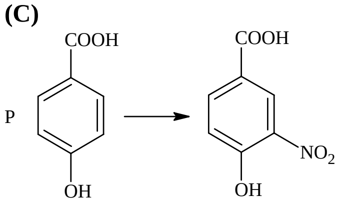
(A)


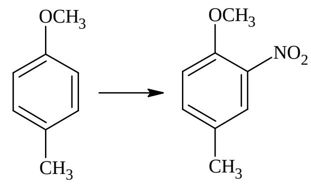
(B)


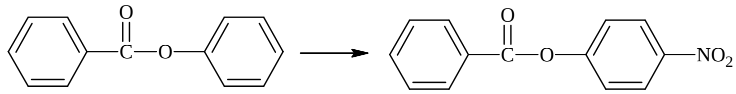
(C)


(D)


Key: **(C)**

Sol.:


Q 

S 

2. Assuming that Hund's rule is violated, the bond order and magnetic nature of the diatomic molecule B_2 is

(A) 1 and diamagnetic (B) 0 and diamagnetic

(C) 1 and paramagnetic (D) 0 and paramagnetic.

Key: **(A)**

Sol.: $B_2 \longrightarrow 10$ electron

$\sigma_{1S}^2 \sigma *_{1S}^2 \ \sigma_{2S}^2 \sigma *_{2S}^2 \ \Pi_{2p_x}^1 \Pi_{2p_y}^1$ (If Hund's rule is obeyed)

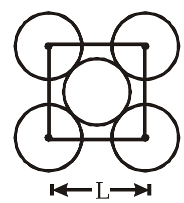
$\sigma_{1S}^2 \sigma *_{1S}^2 \ \sigma_{2S}^2 \sigma *_{2S}^2 \ \Pi_{2p_x}^2 \Pi_{2p_y}^0$ (If Hund's rule is violated)

Bond order = $\dfrac{6-4}{2} = 1$

Paramagnetic if Hund's rule is obeyed

Diamagnetic of Hund's rule is violated.

3. The packing efficiency of the two-dimensional square unit cell shown below is



(A) 39.27% (B) 68.02%

(C) 74.05% (D) 78.54%.

Key: **(D)**

Sol.: Let us consider the squerare plane (given)

Area = L^2

Area covered by $2 \times \Pi r^2$ (two circle)

$4r = \sqrt{2}\,L$

$r = \dfrac{\sqrt{2}}{4} L$

Area = $2 \cdot \Pi \left(\dfrac{\sqrt{2}}{4} L \right)^2 = \Pi L^2 \cdot \dfrac{4}{16} = \dfrac{\Pi L^2}{4}$

Packing efficiency $= \dfrac{\Pi L^2}{4 \times L^2} \times 100$

$= \dfrac{3.14}{4} \times 100 = 78.5\%.$

4. The complex showing a spin-only magnetic moment of 2.82 B.M. is

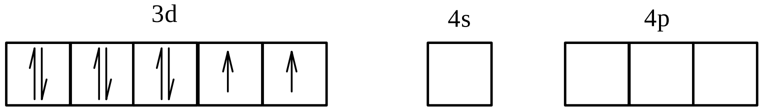
(A) $Ni(CO)_4$ (B) $[NiCl_4]^{2-}$

(C) $Ni(PPh_3)_4$ (D) $[Ni(CN)_4]^{2-}$

Key: **(B)**

Sol.: Ni_{28} — $4S^2 3d^8$

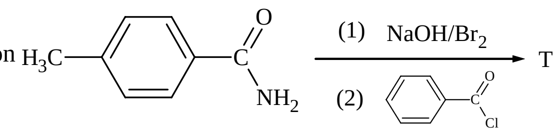
Ni^{+2} — $4S^0 3d^8$

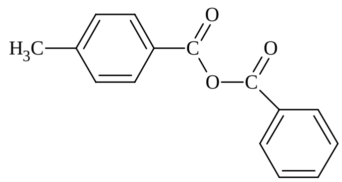


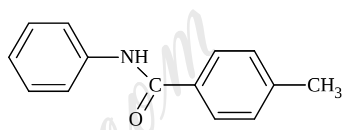
2 unpaired electron (n = 2)

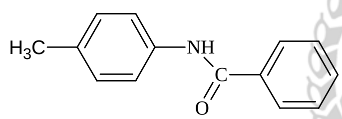
$\mu = \sqrt{n(n+2)} = \sqrt{8}$ BM

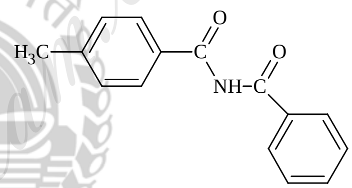
= 2.82 BM.

5. In the reaction  T the structure of the Product T is

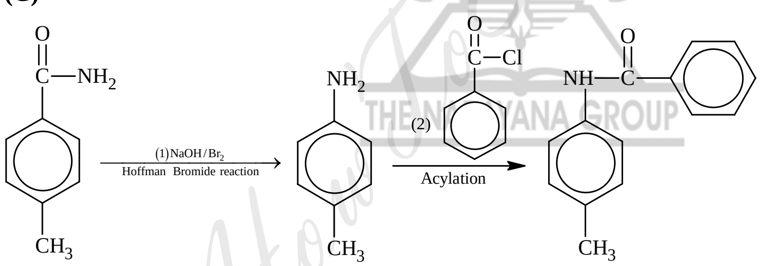
(A) 

(B) 

(C) 

(D) 

Key: **(C)**

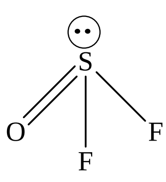
Sol.: 

6. The species having pyramidal shape is

(A) SO_3 (B) BrF_3

(C) SiO_3^{2-} (D) OSF_2

Key: **(D)**

Sol.: 

Pyramidal

SECTION – II
Integer Type

This section contains a group of 5 questions. The answer to each of the questions is a single-digit integer. ranging from 0 to 9. The correct digit below the question no. in the ORS is to be bubbled.

7. Silver (atomic weight = 108 g mol^{-1}) has a density of 10.5 g cm^{-3}. The number of silver atoms on a surface of area 10^{-12} m^2 can be expressed in scientific notation as $y\times10^x$. The value of x is

Key: **(7)**

Sol.: Consider a single layer square shaped arrangement of n $\times$ n silver atoms. Also assume the radius of each silver atom r.

Area of the layer = $(2rn)^2 = 10^{-12}m^2 = 10^{-8}cm^2$

Mass of the layer

Surface area $\times$ thickness $\times$ density = number of atoms $\times$ mass of single atom.

$10^{-8}\times2r\times10.5 = n^2\times108\times1.66\times10^{-24}$ [put 2rn = 10^{-4}]

$\therefore$ $n^3 = 5.855\times10^{10}$

$n \simeq 3.82\times10^3$

$\therefore$ number of silver atoms on the surface

$= n^2 = \left(3.82\times10^3\right)^2$

$= 1.4592\times10^7$

$\therefore$ x = 7.

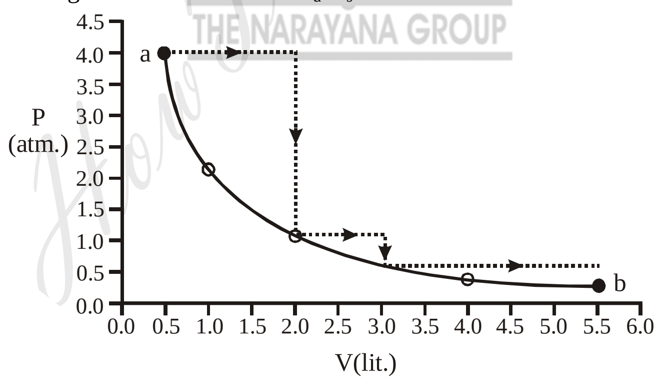
8. Among the following, the number of elements showing only one non-zero oxidation state is

O, Cl, F, N, P, Sn, Tl, Na, Ti

Key: **(2)**

Sol.: Fluorine and sodium shown only one non zero oxidation state, fluorine show – 1 and sodium shown + 1.

9. One mole of an ideal gas is taken from a to b along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is w_s and that along the dotted line path is w_d, then the integer closest to the ratio w_d/w_s is



Key: **(2)**

Sol.: $w_d = 4\times1.5 + 1\times15 + 0.80\times2.5$

$= 9.375$

$w_s = 2.33\ pV\log\frac{v_2}{v_1}$

$= 2.33\times4\times0.5\log\frac{5.5}{0.5}$

≈ 4.606

$$\frac{w_d}{w_s} = \frac{9.375}{4.606} \approx 2 .$$

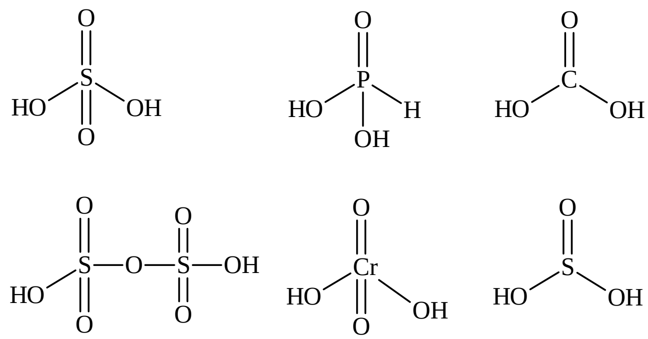
10. The total number of diprotic acids among the following is

H_3PO_4 H_2SO_4 H_3PO_3 H_2CO_3 $H_2S_2O_7$

H_3BO_3 H_3PO_2 H_2CrO_4 H_2SO_3

Key: **(6)**

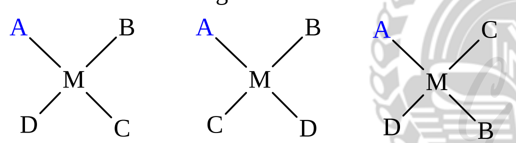
Sol.: The total number of diprotic acids are 6



11. Total number of geometrical isomers for the complex $[RhCl(CO)(PPh_3)(NH_3)]$ is

Key: **(3)**

Sol.: The total number of geometrical isomers for the complex $[RhCl(O)(PPh_3)(NH_3)]$ is 3.



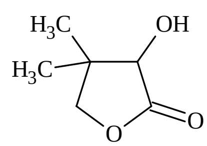
SECTION – III

Paragraph Type

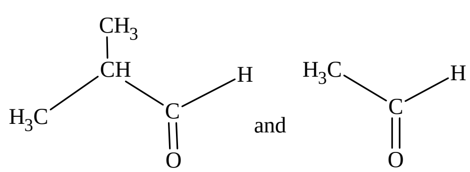
This section contains 2 paragraphs. Based upon each of the paragraphs 3 multiple choice questions have to be answered. Each of these question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

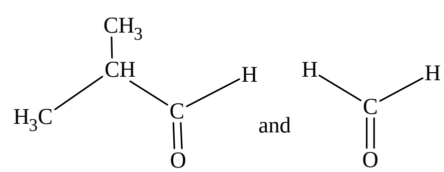
Paragraph for Questions 12 to 14

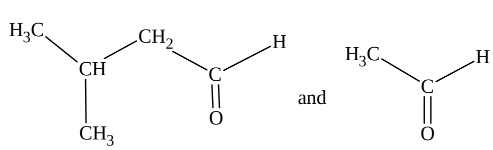
Two aliphatic aldehydes P and Q react in the presence of aqueous K_2CO_3 to give compound R, which upon treatment with HCN provides compound S. On acidification and heating, S gives the product shown below :

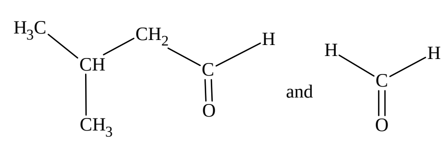


12. The compounds P and O respectively are :

(A) 

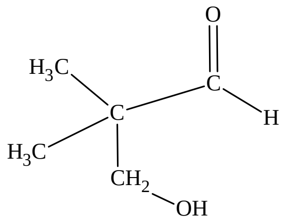
(B) 

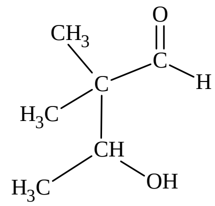
(C) 

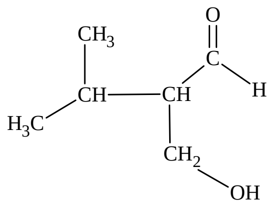
(D) 

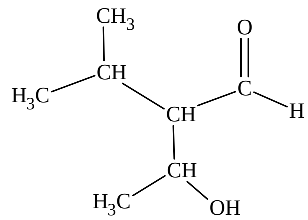
Key: **(B)**

13. The compound R is

(A) 

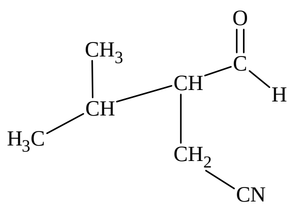
(B) 

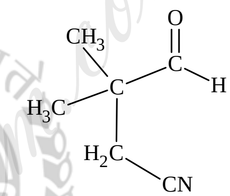
(C) 

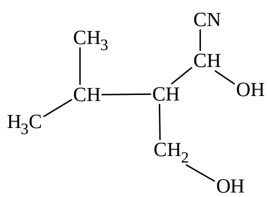
(D) 

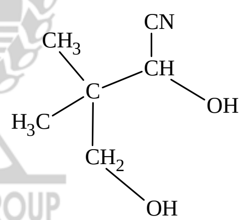
Key: **(A)**

14. The compounds S is

(A) 

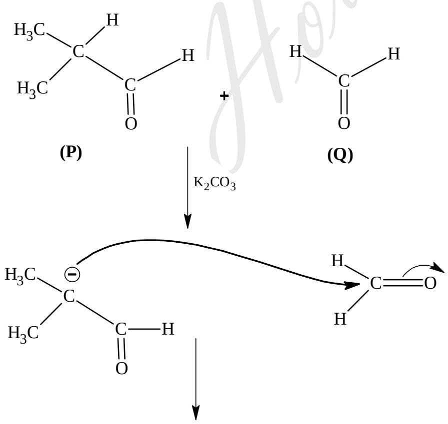
(B) 

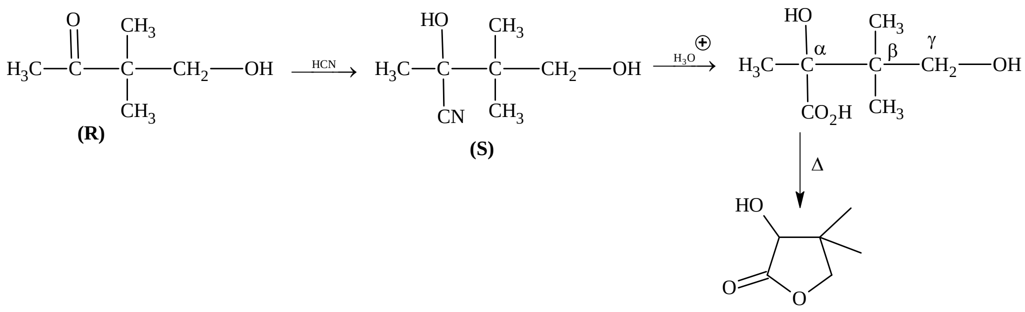
(C) 

(D) 

14. **(D)**

Sol.:(12-14)





Paragraph for Questions 15 to 17

The hydrogen-like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

15. The state S_1 is

(A) 1s (B) 2s

(C) 2p (D) 3s

Key: **(B)**

Sol.: No. of radial node = $n - \ell - 1$

Since state S_1 has 1 radial node it must be 2s orbital with n = 2 and $\ell = 0$.

$\therefore$ (B)

16. Energy of the state S_1 in units of the hydrogen atom ground state energy is

(A) 0.75 (B) 1.50

(C) 2.25 (D) 4.50

Key: **(C)**

Sol.: Energy of state S_1 = $-\frac{13.6}{2^2}\times 3^2$ eV/atom

$= -13.6\times 2.25$ eV / atom

= 2.25 times energy of ground state of hydrogen atom.

$\therefore$ (C)

17. The orbital angular momentum quantum number of the state S_2 is

(A) 0 (B) 1

(C) 2 (D) 3

Key: **(B)**

Sol.: Energy of S_2 level = –13.6 eV/atom

$$-\frac{13.6\times 3^2}{n^2} = -13.6$$

$\therefore$ P.Q.N of level S_2 = 3

Since S_2 has one radial node being present in 3^{rd} shell it must be 3p orbital.

$\therefore$ The orbital angular momentum quantum number

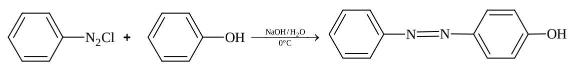
i.e., Azimuthal quantum number of S_2 = 1.

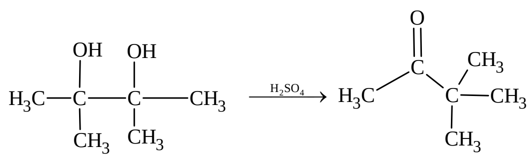
SECTION – IV
Matrix Type

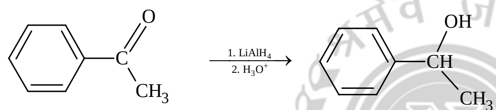
*This section contains **2 questions**. Each question four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.*

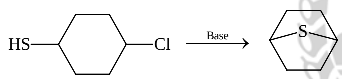
18. Match the reactions in **Column I** with appropriate options in **Column II**.

Column I

(A) 

(B) 

(C) 

(D) 

Column II

(p) Racemic mixture

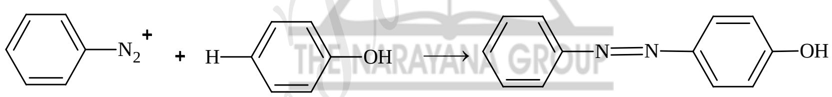
(q) Addition reaction

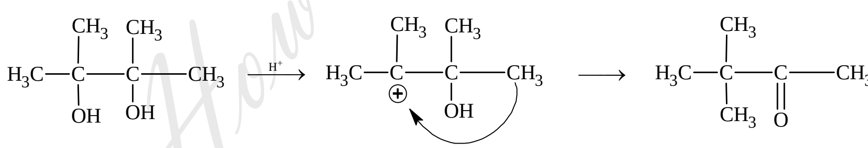
(r) Substitution reaction

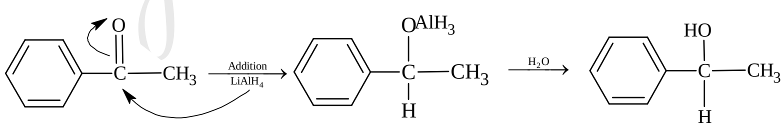
(s) Coupling reaction

(t) Carbonation intermediate

Key: **(A – r, s), (B –t), (C – p, q), (D – r)**

Sol.: (A) 

(B) 

(C) 

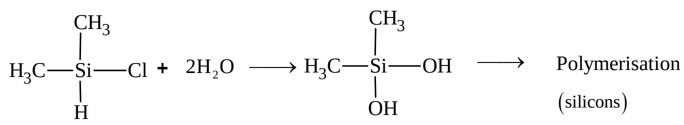
(D) Intramolecular substitution reaction

19. All the compounds listed in **Column I** react with water. Match the result of the respective reactions with the appropriate options listed in **Column II**.

Column I	**Column II**
(A) $(CH_3)_2SiCl_2$	(p) Hydrogen halide formation
(B) XeF_4	(q) Redox reaction
(C) Cl_2	(r) Reacts with glass
(D) VCl_5	(s) Polymerization
	(t) O_2 formation

Key: **(A – p, s), (B –p, q, r), (C – p, q), (D – p)**

Sol.: (A)



(B) $2XeF_4 + 3H_2O \longrightarrow Xe + XeO_3 + 3H_2F_2 + F_2$

$SiO_2 + 4HF \longrightarrow SiF_4 + 2H_2O$

$SiF_4 + 2HF \longrightarrow H_2SiF_6$

(C) $Cl_2 + H_2O \longrightarrow HOCl + HCl.$

(D) $VCl_5 + H_2O \longrightarrow$ must form HCl on hydrolysis

PART – II: MATHEMATICS

SECTION – I

Single Correct Choice Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

20. If the distance of the point P(1, –2, 1) from the plane x + 2y – 2z = α, where α > 0, is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Key **(A)**

Sol. $|5 + \alpha| = 15$

$\Rightarrow \alpha = 10$

If (x_1, y_1, z_1) is foot of perpendicular $\frac{x_1 - 1}{1} = \frac{y_1 + 2}{2} = \frac{z_1 - 1}{-2} = \frac{-(-15)}{9}$

$\therefore (x_1, y_1, z_1) \equiv (8/3, 4/3, -7/3)$

21. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$

(C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Key **(C)**

Sol. $E_1 \to$ original signal is green.

$E_2 \to$ original signal is red.

$E \to$ signal received at station B is green.

$$P(E_1/E) = \frac{p(E_1)\,p(E)/E_1)}{p(E_1)\,p(E/E_1) + p(E_2)\,p(E/E_2)}$$

$$= \frac{\frac{4}{5}\left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]}{\frac{4}{5}\left(\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right) + \frac{1}{5}\left[\frac{3}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{3}{4}\right]} = \frac{20}{23}$$

22. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$

(C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Key **(B)**

Sol. $\overrightarrow{AB}.\overrightarrow{AD} > 0$,

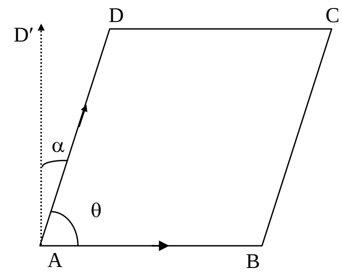
So θ is acute.

$\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|} = \frac{8}{9}$

$\cos(\alpha + \theta) = 0 \Rightarrow \cos\alpha.\cos\theta - \sin\alpha.\sin\theta = 0$

$8\cos\alpha - \sqrt{17}.\sin\alpha = 0$

$64\cos^2\alpha = 17\sin^2\alpha \Rightarrow \cos\alpha = \frac{\sqrt{17}}{9}$



23. For r = 0, 1, ... , 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then

$\sum_{r=1}^{10} A_r\,(B_{10}B_r - C_{10}A_r)$ is equal to

(A) $B_{10} - C_{10}$ (B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$

(C) 0 (D) $C_{10} - B_{10}$

Key **(D)**

Sol. $A_r = {}^{10}C_r$

$B_r = {}^{20}C_r$

$C_r = 30C_r$

$\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$

$= B_{10}\sum_{r=1}^{10} A_rB_r - C_{10}\sum_{r=1}^{10} A_r^2$

$= B_{10}\left[\sum_{r=1}^{10} {}^{10}C_r.{}^{20}C_r - C_{10}\sum_{r=1}^{10}\left({}^{10}C_r\right)^2\right]$

$= B_{10}\,[{}^{10}C_1\,{}^{20}C_1 + {}^{10}C_2\,{}^{20}C_2 \ldots. + {}^{10}C_{10}\,{}^{20}C_{10}] - C_{10}\sum_{r=1}^{10}\left({}^{10}C_r\right)^2$

$= B_{10}.[{}^{30}C_{10} - 1] - C_{10}\,[{}^{20}C_{10} - 1]$

$= C_{10} - B_{10}$

24. Let f be a real-valued function defined on the interval (-1, 1) such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$, for all $x \in (-1, 1)$, and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

(A) 1 (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{1}{e}$

Key (B)

Sol. $\because f(f^{-1}(x)) = x$

$f'(f^{-1}(x)) \cdot (f^{-1}(x))' = 1$

$(f^{-1}(x))' = \dfrac{1}{f'(f^{-1}(x))}$

We have to find $(f^{-1}(2))'$

$(f^{-1}(2))' = \dfrac{1}{f'(f^{-1}(2))}$

When $f^{-1}(x) = 2 \Rightarrow f(x) = 2 \Rightarrow x = 0$

given $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt$

$e^{-x}(-f(x) + f'(x)) = x^4 + 1$

put x = 0

$\Rightarrow f'(0) = 3$

$\Rightarrow (f^{-1}(2))' = 1/3.$

25. Let S = {1, 2, 3, 4). The total number of unordered pairs of disjoint subsets of S is equal to

(A) 25 (B) 34

(C) 42 (D) 41

Key **(D)**

Sol. S = {1, 2, 3,4}, then No. of unordered pairs of disjoint subsets of S is

$\dfrac{3^4+1}{2} = 41$

SECTION – II

Integer Type

This section contains a group of 5 questions. The answer to each of the questions is a single-digit integer. ranging from 0 to 9. The correct digit below the question no. in the ORS is to be bubbled.

26. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ... , 11.

If $\dfrac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\dfrac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

Key **(0)**

Sol. $a_1^2 + a_2^2 + a_3^2 + \ldots + a_{11}^2 = 990$

$\Rightarrow a^2 + (a+d)^2 + (a+2d)^2 + \ldots + (a+10d)^2 = 990$

$\Rightarrow 11a^2 + d^2(1^2 + 2^2 + 3^2 + \ldots + 10^2) + ad(2 + 4 + 6 + \ldots + 20) = 990$

$\Rightarrow 11 \times 225 + d^2 \times 385 + d \times 15 \times 110 = 990$

$\Rightarrow 7d^2 + 30d + 27 = 0$

$\Rightarrow d = -3, -9/7$ (n.p.)

$\therefore d = -3$ and $a_1 = 15$

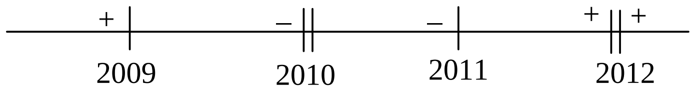
$\therefore \dfrac{a_1 + a_2 + \ldots + a_{11}}{11} = \dfrac{11}{2\times 11}(2\times 15 + 10\times(-3)) = 0$

27. Let f be a function defined on R (the set of all real numbers) such that $f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x \in R$.

If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ln(g(x))$, for all $x \in R$, then the number of points in R at which g has a local maximum is

Key **(1)**

Sol. The sign scheme of $f'(x)$



The local maximum of f(x) occurs at x = 2009

Hence, local maximum of g(x) also occurs at x = 2009. Hence the number of point of local maximum = 1.

28. Let k be a positive real number and let $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$. If det (adj A) + det (adj B) = 10^6, then [k] is equal to

{Note: adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

Key **(4)**

Sol. $|A| = (2k-1)(-1+4k^2) + 2\sqrt{k}\,(2\sqrt{k} + 4k\sqrt{k}) + 2\sqrt{k}\,(4k\sqrt{k} + 2\sqrt{k})$

$(2k-1)(4k^2-1) + 4k + 8k^2 + 8k^2 + 4k$

$= (2k-1)(4k^2-1) + 8k + 16k^2$

$= 8k^3 - 4k^2 - 2k + 1 + 8k + 16k^2 = 8k^3 + 12k^2 + 6k + 1$

|B| = 0 as B is skew symmetric matrix of odd order.

$\Rightarrow (8k^3 + 12k^2 + 6k + 1)^2 = (10^3)^2$

$\Rightarrow (2k+1)^3 = 10^3$

$\Rightarrow 2k + 1 = 10$

$\Rightarrow k = 4.5$

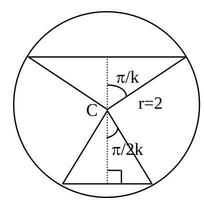
[k] = 4.

29. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is

[Note : [k] denotes the largest integer less than or equal to k]

Key **(3)**

Sol. $2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k} = \sqrt{3}+1$



$2\cos^2\frac{\pi}{2k} + \cos\frac{\pi}{2k} = \frac{3+\sqrt{3}}{2}$

$\cos\frac{\pi}{2k} = \frac{-1 \pm \sqrt{(2\sqrt{3}+1)^2}}{4} = \frac{\sqrt{3}}{2}$ or $\frac{-\sqrt{3}-1}{2}$

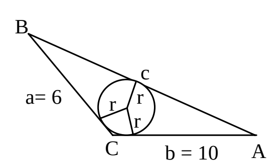
But $\cos\frac{\pi}{2k} \neq \frac{-\sqrt{3}-1}{2}$

$\cos\frac{\pi}{2k} = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow k = 3.$

30. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If ∠ACB is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Key **(3)**

Sol. The area = $15\sqrt{3}$



$\therefore \frac{1}{2} \times 6 \times 10\ \sin C = 15\sqrt{3}$

C = 120°

$\cos C = \dfrac{10^2 + 6^2 - c^2}{2 \times 10 \times 6}$

$\Rightarrow -60 = 136 - c^2 \Rightarrow c^2 = 196 \Rightarrow c = 14.$

Since $r = \dfrac{\Delta}{s} = \dfrac{15\sqrt{3}}{(10+6+14)/2} = \dfrac{15\sqrt{3} \times 2}{30} = \sqrt{3}$

$r^2 = 3.$

SECTION – III

Paragraph Type

This section contains 2 paragraphs. Based upon each of the paragraphs 3 multiple choice questions have to be answered. Each of these question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 31 to 33

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$

Let s be the sum of all distinct real roots of f(x) and let t = |s|.

31. The real number s lies in the interval

(A) $\left(-\dfrac{1}{4}, 0\right)$ (B) $\left(-11, \dfrac{-3}{4}\right)$

(C) $\left(-\dfrac{3}{4}, -\dfrac{1}{2}\right)$ (D) $\left(0, \dfrac{1}{4}\right)$

Key **(C)**

Sol. f(x) is constant function

and $f\left(-\dfrac{3}{4}\right) f\left(-\dfrac{1}{2}\right) < 0$

$f'(x) = 12x^2 + 6x + 2 > 0 \ \forall x \in R$

$\Rightarrow$ f(x) has only one real root in $\left(-\dfrac{3}{4}, -\dfrac{1}{2}\right)$

32. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

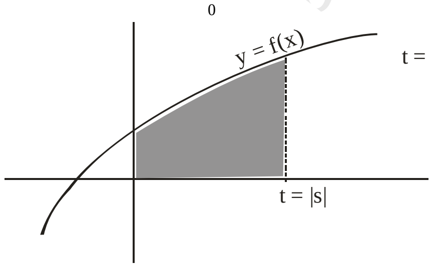
(A) $\left(\dfrac{3}{4}, 3\right)$ (B) $\left(\dfrac{21}{64}, \dfrac{11}{16}\right)$

(C) (9, 10) (D) $\left(0, \dfrac{21}{64}\right)$

Key **(A)**

Sol. Required area

$A = g(t) = \int_0^t f(x)dx$



$t = |s| \in \left(\dfrac{1}{2}, \dfrac{3}{4}\right)$

$= t + t^2 + t^3 + t^4 = \dfrac{t(1-t^4)}{1-t}$

$g'(t) = f(x) > 0 \quad \forall x > 0$

$\Rightarrow$ g(t) is increasing $\forall$ t > 0

$\Rightarrow$ $g\left(\frac{1}{2}\right) < A < g\left(\frac{3}{4}\right)$

$\Rightarrow$ $\frac{15}{16} < A < \frac{525}{256}$ lies in $\left(\frac{3}{4}, 3\right)$

33. The function f′(x) is

(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in (-t, t)

(D) decreasing in (-t, t)

Key **(B)**

Sol. $f'(x) = 12x^2 + 6x + 2$ is increasing

$f''(x) = 24x + 6 > 0$ and $f''(x) < 0$

$x > -\frac{1}{4}$ $x < -\frac{1}{4}$

Hence B is true.

Paragraph for Questions Nos. 34 to 36

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B.

34. The coordinates of A and B are

(A) (3, 0) and (0, 2)

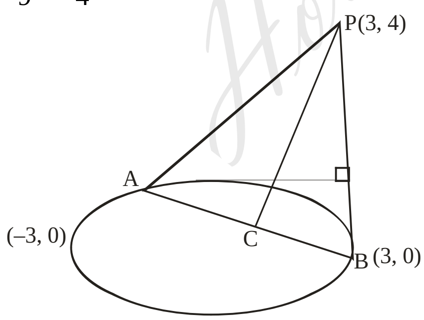
(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Key **(D)**

Sol. $\frac{x^2}{9} + \frac{y^2}{4} = 0$ …..(i)



Equation of chord
Contact AB
x + 3y – 3 = 0 …..(ii)
Solve (i) & (ii) we get

$A = \left(-\frac{9}{5}, \frac{8}{5}\right)$ B(3, 0)

35. The orthocenter of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$

(C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

Key **(C)**

Sol. Equation of PE

$y - 4 = 3(x - 3)$ …..(i)

Equation of AD

$y = \frac{8}{5}$ …..(ii)

Solving (i) & (ii) we get $x = \frac{11}{5},\ y = \frac{8}{5}$

36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ (B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ (D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Key **(A)**

Sol. $\sqrt{(\alpha-3)^2 + (\beta-4)^2} = \frac{(\alpha+3\beta-3)^2}{\sqrt{10}}$

$10(\alpha^2 - 6\alpha + 9 + \beta^2 - 8\beta + 16) = \alpha^2 + 9\beta^2 + 9 + 6\alpha\beta - 6\alpha - 18\beta$

Required locus is $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

SECTION – IV
Matrix Type

*This section contains **2 questions**. Each question four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.*

37. Match the statements in **Column-I** with those in **Column-II.**

[Note: Here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

	Column I		Column II
(A)	The set of points z satisfying $\|z - i\|z\|\| = \|z + i\|z\|\|$ is contained in or equal to	(p)	an ellipse with eccentricity 4/5
(B)	The set of points z satisfying $\|z + 4\| + \|z - 4\| = 10$ is contained in or equal to	(q)	the set of points z satisfying Im z = 0
(C)	If $\|w\| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r)	the set of points z satisfying $\|\text{im } z\| \le 1$
(D)	If $\|w\| = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to	(s)	the set of points z satisfying $\|\text{Re } z\| \le 2$
		(t)	the set of points z satisfying $\|z\| \le 3$

Key. **(A-q, r), (B-p), (C-p, s, t), (D-q, r, s, t)**

Sol. $|z - i|z|| = |z + i|z||$

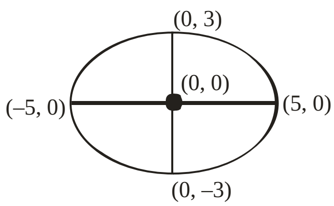
(A) Putting $z = x + iy$

We get $y\sqrt{x^2 + y^2} = 0$

i.e., Im (z) = 0.

(B) $2ae = 8$, $2a = 10$

$10e = 8 \Rightarrow \quad e = \frac{4}{5}$



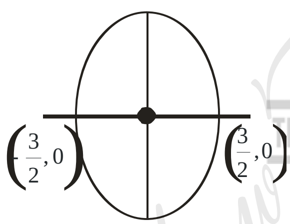
$b^2 = 25\left(1 - \frac{16}{25}\right) = 9$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

(C) $z = 2(\cos\theta + i\sin\theta) - \frac{1}{2(\cos\theta + i\sin\theta)}$

$= 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta - i\sin\theta)$

$z = \frac{3}{2}\cos\theta + \frac{5}{2}i\sin\theta$



Let $z = x + iy$

$x = \frac{3}{2}\cos\theta, \qquad y = \frac{5}{2}\sin\theta$

$\Rightarrow \quad \left(\frac{2x}{3}\right)^2 + \left(\frac{2y}{5}\right)^2 = 1$

$\frac{x^2}{\frac{9}{4}} + \frac{y^2}{\frac{25}{4}} = 1$

$\frac{9}{4} = \frac{25}{4}(1 - e^2)$

$e^2 = 1 - \frac{9}{25} = \frac{16}{25}$

$\Rightarrow \quad e = \frac{4}{5}$

(D) Let $w = \cos\theta + i\sin\theta$

$$z = x + iy = w + \frac{1}{w}$$

$$\Rightarrow \quad x + iy = 2\cos\theta$$

$$x = 2\cos\theta,\ y = 0$$

(q), (s)

38. Match the statements in Column – I with the values in Column-II

	Column I		**Column II**
(A)	A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p)	-4
(B)	The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q)	0
(C)	Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b} = 0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\|\vec{b}+\vec{c}\|=\|\vec{b}-\vec{a}\|$. If $\vec{a} = \mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r)	4
(D)	Let f be the function on [-π, π] given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right)/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(s)	5
		(t)	6

Key. **(A – t), (B - p, r), (C - q), (D - r)**

Sol. **(A)** $P(\lambda+2, 1-2\lambda, \lambda-1)$

$$Q\left(2\mu+\frac{8}{3}, -\mu-3, \mu+1\right)$$

$$|PQ|^2 = d^2 = \left(\lambda - 2\mu - \frac{2}{3}\right)^2 + (\mu - 2\lambda + 4)^2 + (\lambda - \mu - 2)^2$$

As $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are collinear $2\mu + \frac{8}{3} = \frac{1-2\lambda}{-\mu-3} = \frac{\lambda-1}{\mu+1}$

(from last two)

$$\lambda\mu - \lambda + 2 = 0 \qquad \ldots(i)$$

and $$\lambda\mu - 4\mu + \frac{5}{3}\lambda = \frac{14}{3} \text{ (from Ist and IIIrd)} \qquad \ldots(ii)$$

from (i) and (ii) $2\lambda - 3\mu = 5$ . . . (iii) from (i) and (iii) $3\mu^2 + 2\mu - 1 = 0$

$\therefore\ \mu = -1,\ \frac{1}{3}$

so, $\lambda = 1, 3$

Hence, $d^2 = \frac{109}{9}$ or 6

(B) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$

$\Rightarrow \quad \tan^{-1}(x+3) - \tan^{-1}(x-3) = \tan^{-1}\left(\frac{3}{4}\right)$

Let $\tan^{-1}(x+3) = \alpha$, $\tan^{-1}(x-3) = \beta$

$\Rightarrow \tan\alpha = x+3,\ \tan\beta = x-3$

$\tan(\alpha - \beta) = \frac{3}{4}$

$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{3}{4}$

$= \frac{(x+3)-(x-3)}{1+x^2-9} = \frac{3}{4}$

$= \frac{6}{x^2-8} = \frac{3}{4}$

$= x^2 - 8 = 8$

$= x^2 = 16$

$x = \pm 4$

(C) $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$

Put $\vec{C} = \frac{\vec{a} - \mu\vec{b}}{4}$

$(\vec{b} - \vec{a}).\left(\vec{b} + \frac{\vec{a} - \mu\vec{b}}{4}\right) = 0$

$(\vec{b} - \vec{a}).((4-\mu)\vec{b} + \vec{a}) = 0$

$(4-\mu)|b|^2 - |a|^2 = 0$

$\vec{a}.\vec{b} = 0$

$(4-\mu)|b|^2 = |a|^2 \ \ (\because\ \vec{a}.\vec{b} = 0)$

Also $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$ again put $\vec{C} = \frac{\vec{a} - \mu\vec{b}}{4}$

$2\left|\vec{b} + \frac{\vec{a} - \mu\vec{b}}{4}\right| = |\vec{b} - \vec{a}|$

$\frac{1}{2}|(4-\mu)\vec{b} + \vec{a}| = |\vec{b} - \vec{a}|$

$(4-\mu)^2|b|^2+|a|^2=4|b|^2+4|a|^2 \qquad \text{since } \vec{a}.\vec{b}=0$

$\left((4-\mu)^2-4\right)|b|^2=3|a|^2$

$\left((4-\mu)^2-4\right)|b|^2=3(4-\mu)|b|^2$

$(4-\mu)^2-4=12-3\mu$

$16+\mu^2-8\mu-4=12-3\mu$

$\mu^2-5\mu=0$

$\mu = 0$ or 5. but $\mu = 5$ is not satisfying so $\mu = 0$.

(D) $\dfrac{2}{\pi}\displaystyle\int_{-\pi}^{\pi}\dfrac{\sin\left(\frac{9x}{2}\right)}{\sin\frac{x}{2}}$

$$=\frac{4}{\pi}\int_0^{\pi}\frac{2\sin\left(\frac{9x}{2}\right)\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\underbrace{\frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x}{\sin x}}_{I_1}+\underbrace{\frac{4}{\pi}\int_0^{\pi}\frac{\sin 4x}{\sin x}}_{I_2}$$

$I_1=\pi,\ I_2=0$

So, $\dfrac{4}{\pi}\times\pi=4$.

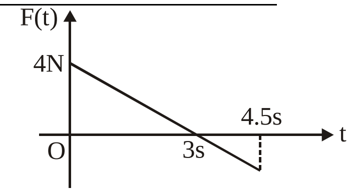
PART – III: PHYSICS

SECTION – I

Single Correct Choice Type

This section contains 6 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

39. A block of mass 2 kg is free to move along the x–axis. It is at rest and from t = 0 onwards it is subjected to a time–dependent force F(t) in the x direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(A) 4.50 J (B) 7.50 J
(C) 5.06 J (D) 14.06 J.

Key. **(C)**

Sol.

$$2(V-0)=\frac{1}{2}\times 4\times 3-\frac{1}{2}\times 2\times 1.5$$

$$2V=6-1.5$$

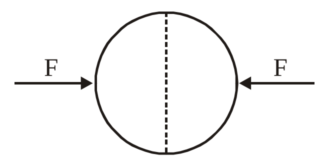
$$V=\frac{4.5}{2}$$

$$K=\frac{1}{2}(2)\left(\frac{9}{4}\right)^2$$

$$=\frac{81}{16}=5.06\text{ J}.$$

Hence correct option is (C).

40. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to

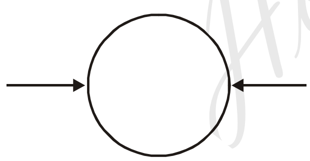


(A) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2R$

(C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$.

Key. **(A)**

Sol.



$$\frac{\sigma^2}{2\varepsilon_0}=\frac{F}{\pi R^2}$$

$$F=\frac{\sigma^2\pi R^2}{2\varepsilon_0}.$$

Hence correct option is (A).

41. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given g = 9.8 ms^{-2}, viscosity of the air = 1.8×10^{-5} Ns m^{-2} and the density of oil = 900 kg m^{-3}, the magnitude of q is

(A) $1.6\times10^{-19}\,C$ (B) $3.2\times10^{-19}\,C$
(C) $4.8\times10^{-19}\,C$ (D) $8.0\times10^{-19}\,C$.

Key. **(D)**

Sol. mg = f_{vise}

$$\frac{4}{3}\pi R^3 \rho g = 6\pi\eta R v_T$$

$$R = \sqrt{\frac{9}{2}\cdot\frac{\eta\cdot v_T}{\rho g}} = \frac{3}{7}\times10^{-5}\,m$$

Eq = mg

$$q = \frac{4}{3}\frac{\pi R^3 \rho g}{E}$$

$$q = 8\times10^{-19}\,C.$$

42. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is

(A) 0.02 mm (B) 0.05 mm
(C) 0.1 mm (D) 0.2 mm.

Key. **(D)**

Sol. V.C. = 1 div of M.S. – 1 div V.S.

$$= 1 \text{ div of M.S.} - \frac{16}{20}\text{div of M.S.}$$

$$= \frac{4}{20}\text{div of M.S.}$$

$$= 0.2 \text{ mm}.$$

Hence correct option is (D).

43. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

(A) virtual and at a distance of 16 cm from the mirror
(B) real and at a distance of 16 cm from the mirror
(C) virtual and at a distance of 20 cm from the mirror
(D) real and at a distance of 20 cm from the mirror.

Key. **(B)**

Sol. For lens

$$\frac{1}{V} - \frac{1}{-30} = \frac{1}{15}$$

V = 30 cm

For mirror

u = + 20 cm

v = – 20 cm

For lens

u = 10

$$\frac{1}{V} - \frac{1}{10} = \frac{1}{15}$$

$$\frac{1}{V} = \frac{3+2}{30}$$

V = 6 cm from lens = 16 cm from mirror.

Hence correct option is (B).

44. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is

(A) 5 grams (B) 10 grams
(C) 20 grams (D) 40 grams.

Key. **(B)**

Sol.
$$\frac{320}{4\times 0.8}=\frac{2}{2\times 0.5}\sqrt{\frac{50}{\mu}}$$
$$\frac{320}{8\times 0.8}=\sqrt{\frac{50}{\mu}}$$
$$50\times 50=\frac{50}{\mu}$$
$$\mu=\frac{1}{50}\text{kg / m}$$
$$m=\frac{1}{50}\times 0.5\times 1000\text{ gm}$$
$$=10\text{ grams.}$$
Hence correct option is (B).

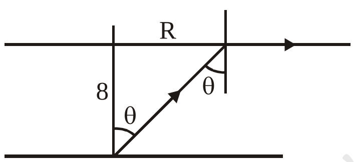
SECTION – II
Integer Type

This section contains a group of 5 questions. The answer to each of the questions is a single-digit integer. ranging from 0 to 9. The correct digit below the question no. in the ORS is to be bubbled.

45. A large glass slab (μ = 5/3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R ?

Key. **6.**

Sol.



$$\sin\theta=\frac{3}{5}\;\Rightarrow\;\tan\theta=\frac{3}{4}$$
$$\frac{R}{8}=\frac{3}{4}\;\Rightarrow\;R=6\text{ cm}.$$

46. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$m to $\frac{50}{7}$m in 30 seconds. What is the speed of the object in km per hour ?

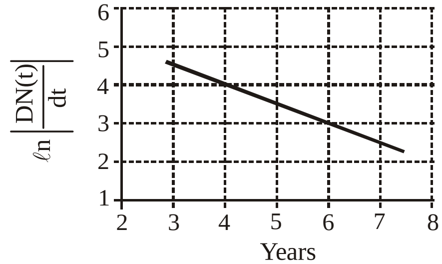
Key. **3.**

Sol.
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$
$$\frac{3}{25}+\frac{1}{u_1}=+\frac{2}{20}$$
$$\frac{1}{u_1}=\frac{+10-12}{100}$$
$$u_1=-50\text{ m}$$
$$\frac{7}{50}+\frac{1}{u_2}=\frac{2}{20}$$
$$\frac{1}{u_2}=\frac{10-14}{100}=-\frac{1}{25}$$

$u_2 = -25$ m

$v = \frac{25}{30} \times \frac{18}{5} = \frac{5 \times 18}{30} = 3$ km / hr .

47. To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is



Key. **8.**

Sol. $\ell n \frac{dN}{dt} = -\frac{1}{2} t$

$\frac{dN}{dt} = e^{-\frac{1}{2}t}$

$\lambda = \frac{1}{2}$ year^{-1}

$T_{1/2} = \frac{0.69}{\left(\frac{1}{2}\right)} = 1.38$ years

4.16 years $\simeq$ 3 half lives

p = 8.

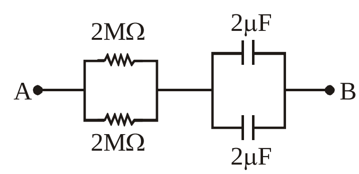
48. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. In the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

Key. **4.**

Sol. $T_i v^{\gamma-1} = \alpha T_i \left(\frac{1}{32} v\right)^{\gamma-1}$

$\alpha = (32)^{\frac{7}{5}t} = (32)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 4$.

49. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V ?
[Take : $\ell n 5 = 1.6$, $\ell n 3 = 1.1$].



Key. **2.**

Sol. $v = v_0(1 - e^{-t/RC})$, $R = \frac{2 \times 2}{2+2} = 10^6 \Omega$, C = 4 × 10^{-6} F

$4 = 10(1 - e^{-t/4})$

$e^{-t/4} = 1.6$

$= \ell n5 - \ell n3 = 0.5$.

$\Rightarrow \frac{t}{4} = 0.5$

$\Rightarrow t = 2s$

SECTION – III
Paragraph Type

This section contains 2 paragraphs. Based upon each of the paragraphs 3 multiple choice questions have to be answered. Each of these question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Question Nos. 50 to 52

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

50. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r << R) is

(A) $2\pi rT$ (B) $2\pi RT$

(C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$.

Key. **(C)**

Sol. $2\pi rT\frac{r}{R} = F$.

51. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, g = 10 ms^{-2}, T = 0.11 Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m

(C) 2.0×10^{-3} m (D) 4.1×10^{-3} m.

Key. **(A)**

Sol.

$$2\pi r^2\frac{T}{R} = mg$$

$$r^2\frac{T}{R} = \rho\frac{4}{3}\pi R^3 g$$

$$\frac{r^2}{R^4}T = \frac{2\rho}{3}g$$

$$R^4 = \frac{(5\times10^{-4})^2\times0.11\times3}{2\times10^4}$$

$$R^4 = \frac{25\times10^{-8}\times0.33}{2\times10^4}$$

$$R = 1.4\times10^{-3}\,\text{m}.$$

52. After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J

(C) 5.4×10^{-6} J (D) 8.1×10^{-6} J.

Key. (B)

Sol.

$$U = 4\pi R^2 s$$

$$= 4\times3.14\times(1.4\times10^{-3})^2\times0.11$$

$$= 2.7\times10^{-6}\,\text{J}.$$

Paragraph for Question Nos. 53 to 55

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

53. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$.

Key. **(D)**

Sol.

$$I\omega = \frac{nh}{2\pi}$$

$$\omega = \frac{nh}{2\pi I}$$

$$K = \frac{1}{2} I \cdot \frac{n^2h^2}{4\pi^2 I^2}$$

$$= \frac{n^2h^2}{8\pi^2 I}.$$

Hence correct option is (D).

54. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}\text{ Hz}$. Then the moment of inertia of CO molecule about its center of mass is close to (Take h = 2π × 10^{-34} J s)

(A) 2.76 × 10^{46} kg m^2 (B) 1.87 × 10^{46} kg m^2

(C) 4.67 × 10^{47} kg m^2 (D) 1.17 × 10^{47} kg m^2.

Key. **(B)**

Sol.

$$\frac{(4-1)h^2}{8\pi^2 I} = h\frac{4}{\pi}\times 10^{11}$$

$$\frac{3h}{32\pi I} = 10^{11}$$

$$\frac{3\times 2\pi \times 10^{-34}}{32 o I} = 10^{11}$$

$$I = 1.87\times 10^{-46}\text{ kg m}^2.$$

55. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where $1\text{a.m.u.} = \frac{5}{3}\times 10^{-27}\text{ kg}$, is close to

(A) 2.4 × 10^{-10} m (B) 1.9 × 10^{-10} m

(C) 1.3 × 10^{-10} m (D) 4.4 × 10^{-11} m.

Key. **(C)**

Sol.

$$1.87\times 10^{-46} = \mu r^2$$

$$1.87\times 10^{-46} = \frac{12\times 16}{28}\times\frac{5}{3}\times 10^{-27} r^2$$

$$r^2 = 1.6\times 10^{-20}$$

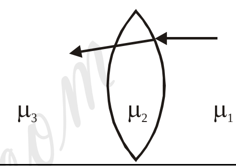
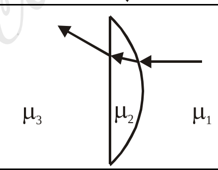
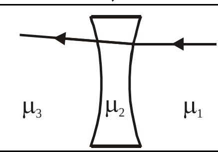
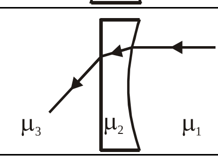
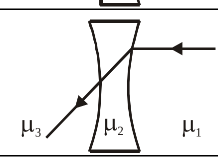
$$r = 1.3\times 10^{-10}\text{ m}.$$

Hence correct option is (C).

SECTION – IV
Matrix Type

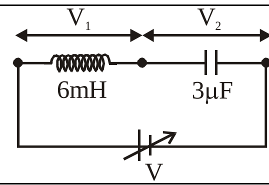
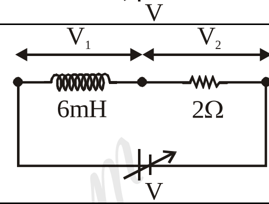
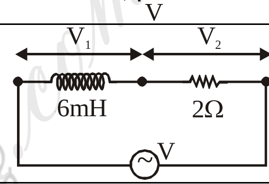
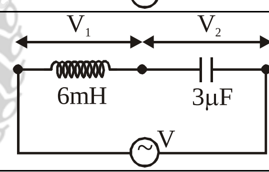
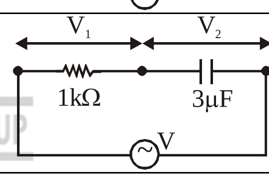
*This section contains **2 questions**. Each question four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.*

56. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationship between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in Column II.

	Column I		**Column II**
(A)	$\mu_1 < \mu_2$	(p)	
(B)	$\mu_1 > \mu_2$	(q)	
(C)	$\mu_2 = \mu_3$	(r)	
(D)	$\mu_2 > \mu_3$	(s)	
		(t)	

Key. **(A) – (p), (r); (B) – (q), (s), (t); (C) – (p), (r), (t); (D) – (q), (s)**

57. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2. (indicated in circuits) are related as shown in Column I. Match the two :

<table>
<tr><th colspan="2">Column I</th><th colspan="2">Column II</th></tr>
<tr><td>(A)</td><td>$I \neq 0$, V_1 is proportional to I</td><td>(p)</td><td>
</td></tr>
<tr><td>(B)</td><td>$I \neq 0$, $V_2 > V_1$</td><td>(q)</td><td>
</td></tr>
<tr><td>(C)</td><td>$V_1 = 0$, $V_2 > V_1$</td><td>(r)</td><td>
</td></tr>
<tr><td>(D)</td><td>$I \neq 0$, V_2 is proportional to I</td><td>(s)</td><td>
</td></tr>
<tr><td></td><td></td><td>(t)</td><td>
</td></tr>
</table>

Key. **(A) – (r), (s), (t); (B) – (q), (r), (s), (t); (C) – (p), (q); (D) – (q), (r), (s), (t),**

Sol.

$$X_L = 2\pi(50)(6\times10^{-3}) = 6\pi\times10^{-1}$$

$$X_C = \frac{1}{2\pi\times50\times3\times10^{-6}} = \frac{106}{3\pi\times100} = \frac{10^4}{3\pi}$$

$$X_C > X_L .$$