B.Tech. (Sem. - $1^{\text {st }} / 2^{\text {nd }}$ )

ENGINEERING MATHEMATICS - II SUBJECT CODE : AM - 102 (New)

Paper ID : [A0119]
[Note : Please fill subject code and paper ID on OMR]

Time : $\mathbf{0 3}$ Hours
Maximum Marks : 60

## Instruction to Candidates:

1) Section - A is Compulsory.
2) Attempt any Five questions from Section - B \& C.
3) Select atleast Two questions from Section - B \& C.

## Section - A

Q1)
(Marks : 2 each)
a) If $A=\left[\begin{array}{lll}2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2\end{array}\right]$ and $B=\left[\begin{array}{lll}1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 2\end{array}\right]$, then the determinant of $A B$ is
(i) 4 ,
(ii) 8 ,
(iii) 16 , (iv)
32
b) The rank of the matrix $A=\left[\begin{array}{ccc}1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3\end{array}\right]$ is $\cdots \cdots$.
c) Two balls of $m_{1}$ and $m_{2}$ gms are projected vertically upward such that the velocity of projection of $m_{1}$ is double that of $m_{2}$. If the maximum height to which $m_{1}$ and $m_{2}$ rise, be $h_{1}$ and $h_{2}$ respectively, then
(i) $h_{1}=2 h_{2}$
(ii) $2 h_{1}=h_{2}$
(iii) $h_{1}=4 h_{2}$
(iv) $4 h_{1}=h_{2}$
d) The complementary part of the differential equation

$$
x^{2} y^{\prime \prime}-x y^{\prime}+y=\log x \text { is } \cdots
$$

(i) $\frac{-x}{2 a} \cos a x$
(ii) $\frac{x}{2 a} \cos a x$
(iii) $\frac{-a x}{2} \cos a x$
(iv) $\frac{a x}{2} \cos a x$.
f) If $u=\left(x^{2}+y^{2}\right)^{-1 / 2}$, then $\nabla \cdot(\nabla u)$ is
(i) 0
(ii) 1
(iii) -1
(iv) 2
g) Maximum value of the directional derivative of

$$
f=x^{2}-2 y^{2}+4 z^{2} \text { at point }(1,1,-1) \text { is } \cdots-\cdots
$$

h) Average scores of three batsman A, B, C are respectively 40, 45, 55 and their standard deviations are respectively $9,11,16$. Which batsman is more consistant?
i) If the correlation coefficient is zero, then regression lines are
(i) parallel
(ii) perpendicular
(iii) coincident
(iv) intersect at $45^{\circ}$.
j) The probability that a leap year should have 53 sundays is
(i) $\frac{2}{7}$
(ii) $\frac{1}{7}$
(iii) 0.3
(iv) 0.5
Section - B

Q2) (a) Find the values of $a, b, c$ if the matrix

$$
\mathrm{A}=\left[\begin{array}{ccc}
0 & 2 b & c \\
a & b & -c \\
a & -b & +c
\end{array}\right]
$$

is orthogonal.
(b) If $\mathrm{A}=\left[\begin{array}{ccc}-1 & 2+i & 5-3 i \\ 2-i & 7 & 5 i \\ 5+3 i & -5 i & 2\end{array}\right]$

Show that A is a Hermitian matrix and $i \mathrm{~A}$ is a skew - Hermitian matrix.

Q3) Solve the following:
(a) $x y\left(1+x y^{2}\right) \frac{d y}{d x}=1$
(b) $\frac{d y}{d x}=\frac{-\left(3 x^{2}+6 x y^{2}\right)}{6 x^{2} y+4 y^{3}}$
(c) $(p x-y)(x+p y)=2 p$.

Q4) Solve the following:
(a) $(\mathrm{D}-2)^{2} y=8\left\{e^{2 x}+\sin 2 x+x^{2}\right\}$.
(b) $x^{3} y^{\prime \prime \prime}+2 x^{2} y^{\prime \prime}+2 y=10\left(x+\frac{1}{x}\right)$.

Q5) (a) Solve

$$
\left(\mathbf{D}^{2}-1\right) y=e^{3 x} \cos 2 x-e^{2 x} \sin 3 x
$$

using method of undetermined coefficients.
(b) Two particles each of mass $m$ gms are suspended from two springs of same stiffness coefficient $k$. After the system comes to rest, the lower mass is pulled $l \mathrm{cms}$ downwards and released. Discuss their motion.


## Section-C

(Marks : 8 each)
Q6) (a) What is conservative field? Show that

$$
\bar{F}=\left(y^{2} \cos x+z^{3}\right) \hat{i}+(2 y \sin x-4) \hat{j}+\left(3 x z^{2}+2\right) \hat{k}
$$

is conservative. Find its scalar potential.


$$
\begin{aligned}
& \int_{\mathrm{S}} \overline{\mathrm{~F}} \cdot d \overline{\mathrm{~S}} \text { where } \\
& \overline{\mathrm{F}}=x^{3} \hat{i}+y^{3} \hat{j}+z^{3} \hat{k}
\end{aligned}
$$

and S is the surface of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$.

Q7) (a) Show that the function $\phi=a \cos m x$ is not a valid velocity potential flow function of liquid.
(b) Test whether the motion specified by

$$
\bar{q}=k^{2}(x \hat{j}-y \hat{i}) /\left(x^{2}+y^{2}\right)(k \text { is constant })
$$

is a possible motion of a liquid.

Q8) (a) Discuss Binomial frequency distribution. The probability that a bomb dropped from a plane hits the target is $\frac{1}{3}$. If 6 bombs are dropped, find the probability that atleast two will hit the target.
(b) The pressure and volume of a gas are related by the equation $p v^{\alpha}=k$, $\alpha$ and $k$ being constants. Find the equation to the following set of values.

| $p\left(\mathrm{~kg} / \mathrm{cm}^{2}\right)$ | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $v$ (litres) | 1.62 | 1.00 | 0.75 | 0.62 | 0.52 | 0.46 |

Q9) (a) Discuss Chi-square test and its properties. Use this to test the hypothesis that data follows a binomial distribution for the problem in which a set of five similar coins is tossed 320 times and the result is

| No. of heads: | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency : | 6 | 27 | 72 | 112 | 71 | 32 |

(b) Two independent samples of size 7 and 6 have the following values:

| Sample A : | 28 | 30 | 32 | 33 | 33 | 29 | 34 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample B : | 29 | 30 | 30 | 24 | 27 | 29 |  |

Examine whether the samples have been drawn from normal populations having the same variance. Given the values of F at $5 \%$ level for 16,57 degrees of freedom is 4.95 and for 15,67 degrees of freedom is 4.39 .

