2057

B.Sc. (HS) in Chemistry II-Semester

CHEMISTRY

Paper—Chem-204

(Mathematics—II)

Time allowed : Three Hours] [Maximum Marks : 75

Note :— Attempt **ALL** questions in Section—A, **EIGHT** questions in Section—B, and any **TWO** questions in Section—C.

SECTION—A

1. Define a continuous function and discuss various types of discontinuities.

2. If f(x) and g(x) are differentiable functions, prove that

$$(f/g)'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}, \; g(x) \neq 0$$

3. Evaluate : $\underset{x \to 0}{\text{Lt}} \left(\frac{1}{x^2} - \cot x \right)$.

4. If a ball, thrown vertically upwards, has equation of motion $s = ut + \frac{1}{2}at^2$ in meters and seconds and if $a = -9{\cdot}8 \text{ m/s}^2$, find the maximum height reached when $u = 30$ m/s.

5. The weight w gm of a liquid in a leaking container is given in terms of time 't' sec. by the relation

$w = 600 - 10t - t^3$. Find the rate at which the liquid is leaking out when $t = 5$ secs.

6. State Rolle's theorem.

7. State Taylor's theorem.

8. Evaluate : $\int \sec 2x \, dx$.

9. Define definite integral and imterpret geometrically.

10. Evaluate : $\int_0^1 \frac{dx}{1+x^2}$. 1½ marks each

SECTION—B

11. Find $\int_0^1 (x^2 - 1)\, dx$ as the limit of sum.

12. Find reduction formula for $I_n = \int \sin^n x \, dx$ and evaluate I_3.

13. Evaluate : $\int \frac{x \tan^{-1} x}{1+x^2} dx$.

14. Evaluate : $\int \frac{x^2 \, dx}{(x-1)(x-2)(x-3)}$.

15. Trace the curve $x^3 + y^3 = 3axy$.

16. Find the Maclaurin's expansion for $\tan^{-1} x$.

17. If $y = \tan^{-1} x$, prove that

$$(1 + x^2)^2 y_2 + 2x(1 + x^2) y_1 = 2.$$

18. Differentiate w.r.t. x :

(i) a^x

(ii) $\sqrt{\dfrac{\sec x + 1}{\sec x - 1}}$

(iii) $\tan^{-1}\left[\sqrt{1+x^2} + x\right]$

19. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, find $\dfrac{dy}{dx}$.

20. Find the area of the region between the curves $y^2 = 4x$ and its latus rectum by using integration.

21. If $u = \tan^{-1}(y/x)$, prove that : $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$.

22. Find the asymptote to the curve :

$y^2(a - x) = x^3$. 4½ marks each

SECTION—C

23. (a) If $\sin y = x \sin(a + y)$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$.

(b) Discuss the maxima and minima of the function :

$$f(x, y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}.$$

24. (a) Show that radius of curvature at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$ is equal to $-\dfrac{3a}{8\sqrt{2}}$.

(b) Evaluate : $\int \frac{x^2+x+1}{(x+1)^2(x+2)}\,dx$.

25. (a) Find the equations of the tangent and normal at $\theta = \frac{\pi}{2}$ to the cycloid

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta).$$

(b) Evaluate : $\int \sqrt{x^2 - 4x + 8}\,dx$.

26. Evaluate :—

(i) $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}$

(ii) $\int \frac{3(x+1)(x+\log x)^2}{x}\,dx$

(iii) $\int_0^{1/2} \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\,dx$.

12 marks each