

2117

B.Sc. (H.S.) Third Semester

CHEMISTRY

Paper—Chem-305

(Mathematics)

Time allowed—Three Hours] [Maximum Marks—75

Note :— Section A is compulsory. Attempt any 8 questions from Section B. Attempt any 2 questions from Section C.

SECTION—A

(each 1½ marks)

1. (a) Define order and degree of the differential equations.
- (b) Form the differential equation, if  $y = e^x(A \cos x + B \sin x)$ .
- (c) Form the partial differential equation from  $z = f(x^2 - y^2)$ .
- (d) Solve :  $r + 6s + 9t = 0$ .
- (e) Write the formula of Trapezoidal rule. Give its order of error also.

- (f) Describe the Horner's method to find the roots of  $f(x) = 0$ .
- (g) What are Dirichlet's conditions ?
- (h) Write the Fourier integral of  $f(x)$ .
- (i) Find the Laplace transform of  $(e^{at} + e^{-at})$ .
- (j) If  $f(t)$  is a periodic function with period  $T$ , then give  $L\{f(t)\}$ .

### SECTION—B

(each  $4\frac{1}{2}$  marks)

2. Solve :

$$(x^2 - y^2)dx - xydy = 0.$$

3. Find particular integral of

$$(D^2 - 2D + 4)y = e^x \cos x.$$

4. The number  $N$  of bacteria in a culture grew at a rate proportional to  $N$ . The value of  $N$  was initially 100 and increased to 332 in one hour. What would be the value of  $N$  after  $1\frac{1}{2}$  hours ?

5. Solve the equation  $y^2z p + x^2z q = y^2x$ .

6. Solve the partial differential equation

$$\frac{\partial^2 y}{\partial t^2} - a^2 \frac{\partial^2 y}{\partial x^2} = E \sin pt.$$

7. Evaluate :

$$\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dx dy dz.$$

8. Find a negative root of  $x^3 - 21x + 3500 = 0$  correct to 2 decimal places using Newton Raphson method.

9. Evaluate  $\int_0^1 \frac{1}{1+x} dx$  taking 6 subintervals by using Simpson's  $\frac{1}{3}$  rule.

10. Express  $f(x) = e^{-x}$  as a Fourier series in the interval  $0 < x < 2\pi$ .

11. Find the Laplace transform of  $\frac{(\cos 2t - \cos 3t)}{t}$ .

12. Using Convolution theorem, evaluate

$$L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\}$$

13. Solve the differential equation

$$\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 5y = e^t \sin t, \text{ where } y(0) = 0 \text{ and } y'(0) = 1, \text{ using Laplace transforms.}$$

### SECTION—C

(each 12 marks)

14. (a) Solve :

$$\frac{d^2y}{dx^2} - 4y = x \sin hx.$$

(b) Solve  $2z + p^2 + qy + 2y^2 = 0$ , using Charpit's method.

15. (a) Show using double integrals that the area between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$  is  $\frac{16}{3}a^3$ .

(b) Find the centre of gravity of the cardioid  $r = a(1 + \cos \theta)$ .

16. Obtain a half range cosine series for  $f(x)$  given by :

$$f(x) = \begin{cases} kx & \text{for } -\pi \leq x \leq 0; \\ x+1 & \text{for } 0 \leq x \leq \pi. \end{cases} \quad \begin{matrix} 8 \\ \cup \end{matrix}$$

Also deduce the sum of the series  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

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17. (a) Solve the simultaneous equations

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t \quad \text{given that } x(0) = 1, \\ y(0) = 0.$$

(b) If  $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots$ , then find the Laplace transform of  $J_0(x)$ .