6563/KAB **OCTOBER 2007**

Optional — GRAPH THEORY AND DATA STRUCTURES

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. Prove that in a connected graph G with exactly 2k odd vertices, there exists k-edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

2. Prove that a graph G is a tree if and only if there is one and only path between every pair of vertices of G.

3. Describe Prim's algorithm for shortest spanning tree.

4. Prove that every circuit has an even number of edges in common with any cutset.

5. Prove that the maximum vertex connectivity in a graph with n vertices and e edges is $\lfloor 2e/n \rfloor$.

6. Define chromatic polynomial of a graph. Illustrate it.

7. Prove that a covering g of a graph is minimal if and only if I contains no paths of length three or more.

8. State and prove the path-length theore.

SECTION B — (3 × 20 = 60 marks)

Answer any THREE questions

9. Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree.

10. (a) Explain traveling salesman problem.

(b) Prove that a tree with n vertices has n-1 edges.

11. (a) Prove that the ring sum of two cutsets in a graph is either a third cutset or an edge-disjoint union of cutsets.

(b) Illustrate the result.

12. (a) State and prove Euler's formula for connected graphs.

(b) Deduce that K_5 and $K_{3,3}$ are nonplanar.

13. State and prove five colour theorem.

14. (a) If T is a 2-tree with k leaves, prove that the minimum values for h and E(T) occur when all the leaves of T are on the same level or on two adjacent leaves.

(b) Explain the method of sequential search.