6560/KA5 OCTOBER 2007

Paper V — MATHEMATICAL STATISTICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (4 × 10 = 40 marks)

Answer any FOUR questions.

1. State and prove Chebyshev's inequality.

2. With usual assumptions, prove that X_1 and X_2 are stochastically independent if and only if $M(t_1, t_2) = M(t_1, 0)\, M(0, t_2)$.

3. If Y is $b(n, p)$, prove that $\lim_{n \to \infty} P_r\left(\left|\frac{Y}{n} - p\right| < \varepsilon\right) = 1$.

4. Find the mgf, mean and variance of gamma distribution.

5. If $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere,} \end{cases}$ is the pdf of a continuous random variable, find the pdf of $8X^3$.

6. If Y_1, Y_2, Y_3 is the order statistics of a random sample of size 3 from a distribution having pdf $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{elsewhere,} \end{cases}$ find the pdf of $Z_1 = Y_3 - Y_1$.

7. Find the confidence intervals for means with known variance σ^2.

8. If S^2 is the variance of a random sample of size $n > 1$ from a distribution that is $n(\mu, \theta), 0 < \theta < \infty$, what is the efficiency of the estimator $\frac{nS^2}{n-1}$?

SECTION B — (3 × 20 = 60 marks)

Answer any THREE questions.

9. (a) State and prove any four properties of a distribution function.

(b) If X has the mgf $M(t) = e^{\frac{t^2}{2}}, -\infty < t < \infty$, find the expectation of all powers of X.

10. If X and Y are continuous random variables with joint pdf $f(x, y)$,

(a) find the conditional mean of Y, given $X = x$ (if it is linear)

(b) variance of the conditional distribution.

11. If X_1 and X_2 are two samples from a distribution with pdf $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{elsewhere,} \end{cases}$

(a) find the joint pdf of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$

(b) find the marginal pdf of Y_1 and Y_2.

12. (a) Derive Student's t-distribution.

(b) If Z_n is $\chi^2(n)$, prove that $Y_n = (Z_n - n)/\sqrt{2n}$ has a limiting normal distribution with mean 0 and variance 1.

13. (a) State and prove Neyman-Pearson theorem.

(b) If X is $b(n, p)$ and $Y = \frac{X - np}{\sqrt{npq}}$, prove that Y^2 is $\chi^2(1)$ approximately.

14. (a) State and prove Rao-Cramer inequality.

(b) If $X_1, X_2, \cdots, X_n$ is a random sample from a Poisson distribution with mean $\theta > 0$, find an efficient estimator of θ.