(6 pages)

6561/KA6 **OCTOBER 2007**

Paper VI — OPERATIONS RESEARCH

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — ($4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. Use simplex method to solve the following LPP

Maximize $z = 2x_1 + x_2$

Subject to

$$4x_1 + 3x_2 \leq 12$$
$$4x_1 + x_2 \leq 8$$
$$4x_1 - x_2 \leq 8$$
$$x_1, x_2 \geq 0.$$

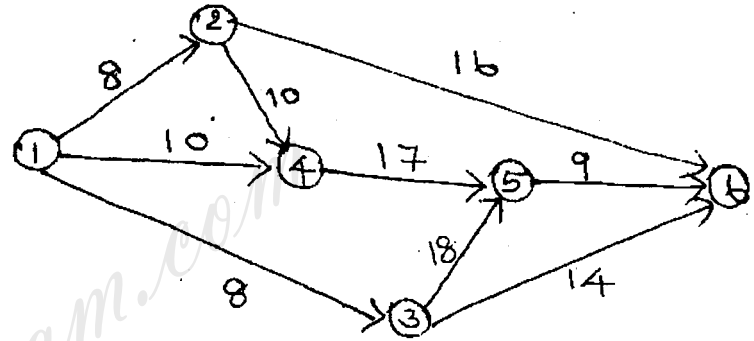
2. Solve the following problem by dual simplex method :

Minimize $z = 20x_1 + 16x_2$

Subject to

$$x_1 + x_2 \geq 12$$
$$2x_1 + x_2 \geq 17$$
$$x_1 \geq 2.5$$
$$x_2 \geq 6$$
$$x_1, x_2 \geq 0.$$

3. For the network given below, find the minimum time of completion of the project. Also identify the critical path.



4. Use dynamic programming to find the value of

Minimize $z = y_1^2 + y_2^2 + y_3^2$

Subject to

$$y_1 + y_2 + y_3 \geq 15$$
$$y_1, y_2, y_3 \geq 0.$$

5. Solve the following 2×5 game by graphic mehtod.

$$\text{Player } A \begin{pmatrix} -5 & 5 & 0 & -1 & 8 \\ 8 & -4 & -1 & 6 & 5 \end{pmatrix}$$

Player B

6. Explain the Branch and Bound method.

7. A branch of National Bank has only one typist. Since the typing work varies in length (number of pages to be typed), the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour. The letter arrive at a rate of 5 per hour during the entire 8-hour work day. If the typewriter is valued at Rs. 1.50 per hour, determine

(a) Average system time

(b) Average idle time cost of the typewriter per day.

8. Find the minimum of the function

$$f(x) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 56.$$

SECTION B — ($3 \times 20 = 60$ marks)

Answer any THREE questions.

9. (a) Describe the role of duality for sensitivity analysis of an L.P. problem.

(b) Consider the problem

Maximize $z = 5x_1 + 3x_2 + 7x_3$

Subject to

$$x_1 + x_2 + x_3 \leq 22$$
$$3x_1 + 2x_2 + x_3 \leq 26$$
$$x_1 + x_2 + x_3 \leq 18$$
$$x_1, x_2, x_3 \geq 0.$$

What will be the solution if the first constraint changes to $x_1 + x_2 + 2x_3 \leq 26$?

10. Solve the following integer programming problem

Maximize $z = 2x_1 + 3x_2$

Subject to

$$6x_1 + 5x_2 \leq 25$$
$$x_1 + 3x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

and integers.

11. A project is represented by the network given below. The activity times are given below.

Activity :	A	B	C	D	E	F	G	H	I
Optimistic time :	5	18	26	16	15	6	7	7	3
Most likely time :	8	20	33	18	20	9	10	8	4
Pessimistic time :	10	22	40	20	25	12	12	9	5

Determine the following :

(a) Expected task times and their variances

(b) The critical path

(c) The probability of completing the project in 41.5 weeks.

12. (a) Explain the Birth-Death process.

(b) A shipping company has a single unloading dock with ships arriving in a Poisson fashion at an average rate of 3 per day. The unloading time distribution for a ship with n unloading crews is found to be exponential with average unloading time $\frac{1}{2n}$ days. The company has a large labour supply without regular working hours and to avoid long waiting times, the company has a policy of using as many unloading crews as there are ships waiting in line or being unloaded. Find

(i) the average number of unloading crews working at any time and

(ii) the probability that more than 4 crews will be needed.

13. Apply Wolfe's method to solve the quadratic programming problem :

Maximize $z = 2x_1 + x_2 - x_1^2$

Subject to

$$2x_1 + 3x_2 \leq 6$$
$$2x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0.$$

14. Use separable programming algorithm to the non-linear programming problem :

Maximize $z = x_1 + x_2^4$

Subject to the constraints :

$$3x_1 + 2x_2^2 \leq 9$$
$$x_1 \geq 0, x_2 \geq 0.$$