

- (b) Find the constants 'a' and 'b' so that the vector $\vec{F} = (axy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (bxz^2 - y)\vec{k}$ is irrotational and find ϕ such that $\vec{F} = \text{grad } \phi$. (7 Marks)
- (c) Evaluate $\int_C xy \, dx + xy^2 \, dy$ by Stokes' theorem where C is the square in the xy plane with vertices (1,0), (-1,0), (0,1) & (0,-1). (6 Marks)

PART - C

5. (a) Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x + 4$ (7 Marks)
- (b) Solve: $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 - 4x - 6$ (7 Marks)
- (c) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2 = e^x \tan x$ using method of variation of parameters. (6 Marks)
6. (a) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$ subject to the conditions $\frac{dy}{dx} = 2, y = 1$ at $x = 0$. (7 Marks)
- (b) Using method of undetermined coefficients solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = x^2 + \cos x$. (7 Marks)
- (c) Solve $(3x - 2)^2 \frac{d^2y}{dx^2} - 3(3x - 2)\frac{dy}{dx} = 9(3x - 2) \sin(\log(3x - 2))$ (6 Marks)

PART D

7. (a) Find the Laplace transforms of
- i) $te^{2t} - 2\frac{\sin 3t}{t}$
- ii) $\int_0^t e^{-t} \sin 2t \sin 3t \, dt$ (7 Marks)
- (b) A periodic function of period $\frac{2\pi}{\omega}$ is defined by
- $$f(t) = \begin{cases} E \sin \omega t, & 0 \leq t \leq \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} \leq t < \frac{2\pi}{\omega} \end{cases}$$
- where E & ω are constants. Show that
- $$L\{f(t)\} = \frac{E\omega}{(s^2 + \omega^2)(1 - e^{-\frac{\pi s}{\omega}})}$$
- (7 Marks)
- (c) Express the following function in terms of Heaviside unit step function & hence find its Laplace transform where
- $$f(t) = \begin{cases} t^2, & 1 < t \leq 2 \\ 4t, & t > 2 \end{cases}$$
- (6 Marks)
8. (a) Find inverse Laplace transform of $\frac{s+1}{(s-1)^2(s+2)}$ (7 Marks)
- (b) Using convolution theorem obtain the inverse Laplace transform of $\frac{s}{(s+2)(s^2+9)}$ (7 Marks)
- (c) Using Laplace transform solve the equation $y'' + 6y' + 9y = 12t^2e^{-3t}$ subject to the conditions $y(0) = 0, y'(0) = 0$. (6 Marks)
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