



IV Semester B.A./B.Sc. Degree Examination, June 2008
(Semester Scheme)
MATHEMATICS (Paper – IV)

Time : 3 Hours

Max. Marks : 90

Instructions : 1) Answer *all* questions.

2) Answers should be written **completely** either in **English** or in **Kannada**.

I. Answer **any fifteen** of the following :

(15×2=30)

- 1) Show that the subgroup $H = \{ 1, -1 \}$ of the multiplicative group $G = \{ 1, -1, i, -i \}$ is normal in G .
- 2) If G is a group and H is a subgroup of index 2 in a group G , show that H is normal subgroup of G .
- 3) Show that every quotient group of an abelian group is an abelian group.
- 4) Define a homomorphism of groups.
- 5) Show that $f : G \rightarrow G$ defined by $f(x) = 2^x$ is not a homomorphism, where G is the multiplicative group of non-zero real numbers.
- 6) Prove that the function $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$ is not continuous at $(0, 0)$.
- 7) State Taylor's theorem for a function of two variables.
- 8) Prove that there is a minimum value at $(0, 0)$ for the function $x^3 + y^3 - 3xy$.
- 9) Prove that $\int_0^{\infty} x^{3/2} e^{-x} dx = \frac{3}{4} \sqrt{\pi}$.
- 10) Prove that $\Gamma(n+1) = n \Gamma(n)$.
- 11) Evaluate $\int_0^{\pi/2} \sqrt{\cot \theta} d\theta$.

P.T.O.



12) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$.

13) Find the particular integral of $(D^2 - 6D + 13)y = 5e^{2x}$.

14) Show that the equation $(1+x^2)y'' + 3xy' + y = 1 + 3x^2$ is exact.

15) Verify the integrability condition of the equation $yz \, dx - 2xz \, dy + (xy - zy^3) \, dz = 0$.

16) Evaluate : $L \{ e^{-t} \sin 2t \}$.

17) Evaluate : $L^{-1} \left\{ \frac{S}{(S+2)^2} \right\}$.

18) Define Heavyside unit function.

19) Find all the basic solutions of the system of linear equations $3x + 2y + z = 22$,
 $x + y + 2z = 9$.

20) Solve graphically the following system of inequalities.

$$2x + y \geq 3, x - 2y \leq -1, y < 3.$$

II. Answer **any two** of the following :

(2×5=10)

1) Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right cosets of H in G is also a right coset of H in G .

2) Prove that a subgroup H of a group G is normal if and only if $ghg^{-1} \in H$, for all $g \in G, h \in H$.

3) If $f: G \rightarrow G'$ is a homomorphism and H is a subgroup of G , then prove that $f(H)$ is a subgroup of G' .

4) Prove that every finite group is isomorphic to a permutation group.

III. Answer **any three** of the following :

(3×5=15)

- 1) Expand $f(x, y) = e^x \cos y$ at the point $(1, \frac{\pi}{4})$ using the Taylor's theorem upto that second degree term.
- 2) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
- 3) Find the extreme values of $x^2 + y^2$ subject to the condition $2x^2 + 3xy + 2y^2 = 1$.
- 4) Prove that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m, n)$ where m and n are both positive.

OR

Show that $\int_0^a x^4 \sqrt{a^2 - x^2} dx = \frac{\pi a^6}{32}$.

- 5) If m is an integer, then prove that $\Gamma(m) \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m+1}} \Gamma(2m)$.

OR

Evaluate $\int_0^2 (4 - x^2)^{3/2} dx$.

IV. Answer **any three** of the following :

(3×5=15)

- 1) Solve $(D^2 + 3D + 2)y = x^2 + \cos x$.
- 2) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2y = x \log x$.
- 3) Solve $x^2 y'' - x(x+2)y' + (x+2)y = x^3 e^x$.
using e^{-x} as a part of complimentary function.
- 4) Solve $(x^2 + 1)y'' - 2xy' + 2y = 6(x^2 + 1)^2$ by the method of variation of parameters.
- 5) Solve $\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$.



V. Answer **any two** of the following :

(2×5=10)

1) If $L \{ f(t) \} = F(s)$, then prove that $L \left\{ \frac{f(t)}{t} \right\} = \int_s^\infty F(S) ds$.

2) i) Evaluate : $L^{-1} \left\{ \frac{S^2}{(S-1)^2} \right\}$.

ii) Prove that $\int_0^\infty te^{-3t} \sin t dt = \frac{3}{50}$.

3) Solve : $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$, given $y(0) = 1$, $y'(0) = 0$ by using Laplace transform method.

VI. Answer **any two** of the following :

(2×5=10)

1) Maximize $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$ subject to the constraints

$$2x_1 + 3x_2 - 4x_3 + 4x_4 = 8, x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

2) The manager of an oil refinery must decide on the optimal mix of two possible blending process of which the inputs and outputs per production run as follows.

Process	Input Units		Output Units	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amounts available of Crude A and B are 200 units and 150 units respectively. Market requirements show that atleast 100 units of gasoline X and 80 units of gasoline Y must be produced. The profit per production run from process 1 and process 2 are Rs. 300 and Rs. 400 respectively. Solve LPP by graphical method.

3) Solve the following L.P.P. by simplex method. Maximize : $f = x - y + 3z$. subject to the constraints $x + y + z \leq 10$, $2x - z \leq 2$, $2x - 2y + 3z \leq 0$, $x, y, z \geq 0$.