Government college of Engineering, Aurangbad

(An autonomous Institute of Government of Maharashtra)

FY MCA Examination

End semester Examination Dec 2011

MCA 104: MATHEMATICS

Time: Three Hours Max. Marks: 60

"Verify the course code and check whether you have got the correct question paper".

N.B.

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Assume suitable data if necessary and state it clearly.
4. Use of non programmable calculator is allowed.

Q.1: Attempt any two of the following (12)

(a) Discuss the convergence of the series

$$\frac{x}{1} + \frac{1}{2}\frac{x^3}{3} + \frac{1}{2}\frac{3}{4}\frac{x^5}{5} + \frac{1}{2}\frac{3}{4}\frac{5}{6}\frac{x^7}{7} + \ldots\ldots (x>0)$$

(b) Examine the convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots\ldots\ldots$$

(c) Discuss the convergence of the infinite series

$$\sum \left(\frac{n}{n+1}\right)^{n^2}$$

Q.2: Attempt any two of the following (12)

(a) Test for the consistency and solve if possible

$x+2y+z=3$

$2x+3y+2z=5$

$3x-5y+5z=2$

$3x+9y-z=4$

(b) Find the Eigen values and Eigen vectors for the following matrix

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

(c) Find the rank of the matrix by reducing it to its normal form

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

Q.3: Attempt any two of the following (12)

(a) Let T:U→V be defined by T(x_1,x_2)=(x_1,0) show that T is a linear map.

(b) Find the matrix representation of each of the mapping T on R^2

Relative to the usual basis {(1,0), (0,1)}

(i) T(*x*, y)=(2*y*, 3*x*-*y)*

(ii) T(*x,y)=(3x-4y, x+5y)*

(c) Determine whether the vectors (1, -2, 3), (5, 6, -1) and (3, 2, 1) are linearly dependent? If so find the relation between them.

Q.4: Attempt any two of the following (12)

(a) Show that the vectors $\left(-\frac{1}{2},\frac{1}{2},0\right)$, $\left(\frac{1}{3},\frac{1}{3},\frac{-2}{3}\right)$ and $\left(\frac{1}{5},\frac{1}{5},\frac{1}{5}\right)$ are orthogonal. Apply Gram-Schmidt process to orthogonalize these vectors.

(b) Find two vectors of norm *1* that are orthogonal to three vectors $u = (2, 1, -4, 0), v = (-1, -1, 2, 2), w = (3, 2, 5, 4)$.

(c) Let V_2 be the vector space with inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$. If $f(x) = \cos(2\pi x), g(x) = \sin(2\pi x)$ show that *f* and *g* are orthogonal.

Q.5: Attempt any two of the following (12)

(a) State whether the quadratic form

$$5x_1^2 + 3x_2^2 + 6x_3^2 + 2x_1x_2 + 3x_2x_3 - 8x_1x_3$$

Is (i) Positive definite (ii) negative definite (iii) semi positive definite (iv) semi negative definite?

(b) Define trace, index and signature.

Find the matrix of the quadratic form

$$3x_1^2 + 4x_2^2 + 5x_3^2 - 5x_1x_2 - 5x_2x_3 + 7x_3x_1.$$

(c) Reduce the quadratic form to the canonical form

$$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx = 2xy$$