(b) Prove that

$$\frac{1 + \tan hx}{1 - \tan hx} = \cos hx + \sin h\, 2x.$$

8. (a) Prove that

$$u = \log_e \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \text{ iff } \cos hu = \sec \theta$$

(b) If

$$i^{x + iy} = A + iB,$$

show that

$$A^2 + B^2 = e^{-(4n + 1)\pi y}.$$

9. (a) Find the equation of the plane containing the point (–1, 7, 2) and the line

$$\frac{x + 3}{2} = \frac{y + 2}{3} = \frac{z - 2}{-2}.$$

(b) Find the equation of the sphere passing through the points (0, 0, 0), (1, 0, 0), (0, 1, 0) and (0, 0, 1)

10. Find the shortest distance between the lines

$$\frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4}$$

$$\frac{x - 2}{3} = \frac{y - 3}{4} = \frac{z - 4}{5}.$$

Register Number :

Name of the Candidate :

1 2 1 6

B.Sc. DEGREE EXAMINATION, 2010

(APPLIED CHEMISTRY / ELECTRONIC SCIENCE / PHYSICS)

(FIRST YEAR)

(PART - III - B - ANCILLARY)

(PAPER - I)

550. MATHEMATICS - I

(Including Lateral Entry)

May] [Time : 3 Hours

Maximum : 75 Marks

Answer any FIVE questions.
All questions carry equal marks.

(5 × 15 = 75)

1. (a) Sum the series to infinity

$$\frac{1 \cdot 2}{3\,!} + \frac{2 \cdot 3}{4\,!} + \frac{3 \cdot 4}{5\,!} + \ldots..$$

Turn over

(b) Find

$$\sum_{n=1}^{\infty} \frac{(n-1)}{(n+2)\ n!} x^n$$

2. (a) Prove that a subgroup of a cyclic group is cyclic.

(b) Let H be a subgroup of index 2 in a group G. Show that H is a normal subgroup of G.

(c) Let G be a group such that $a^2 = e$ for all $a \in G$. Then prove that G is abelian.

3. (a) If

$$y = e^{a \sin^{-1}(x)}$$

prove that

$$(1 - x^2)y_{n+2} - (2n+1)\, xy_{n+1} - (n^2 + a^2)\, y_n = 0.$$

(b) Find the radius of curvature of the curve $r^2 = a^2 \sin 2\theta$.

4. (a) Find the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 4 & 2 \end{pmatrix}.$$

(b) Show that the non - singular matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

satisfies the equation

$$A^2 - 2A - 5I = 0.$$

5. Find the eigen values and the eigen vectors of the matrix

$$\begin{pmatrix} 8 & 2 & -2 \\ 3 & 3 & -1 \\ 24 & 8 & -6 \end{pmatrix}.$$

6. Show that the equations

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

are consistent and solve them.

7. (a) Expand $\sin^7\theta$ in a series of sines of multiples of θ.

Turn over