(b) Evaluate :

$$\lim_{x \to \pi/2} (\sin x)^{\tan x}.$$

9. (a) If

$$u = \sin^{-1}\left(\frac{y}{x}\right),$$

prove that

$$\frac{\partial^2 u}{\partial x\, \partial y} = \frac{\partial^2 u}{\partial y\, \partial x}.$$

(b) If

$$u = e^{x^3 + y^3},$$

prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u \log u.$$

10. (a) Find the sum to infinity of the series

$$\frac{1}{1 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 5} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 7} + \ldots.$$

(b) Find the sum to infinity of the series

$$\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} + \ldots..$$

Register Number :

Name of the Candidate :

5 2 2 9

B.Sc. DEGREE EXAMINATION, 2008

(MATHEMATICS)

(FIRST YEAR)

(PART - III - A - MAIN)

(PAPER - I)

530. ANALYSIS - I

December] [Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.
All questions carry equal marks.

(5 × 20 = 100)

1. (a) State and prove Dedekind's theorem on real numbers.

(b) Prove that (0, 1) is uncountable.

2. (a) Discuss the convergence of the series

$$\Sigma \frac{1}{n^K}.$$

Turn over

(b) Discuss the convergence of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$$

3. (a) Find

$$\frac{dy}{dx}, \text{ if}$$

(i) $y = 3x^2 e^{3x} \cos x$.

(ii) $y = \dfrac{x(x-1)}{x+1}$.

(b) Differentiate

$$\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$$

with respect to $\sqrt{1 - x^2}$.

4. (a) Find the equation of the tangent to the curve $y = x^3$ at the point $\left(\frac{1}{2}, \frac{1}{8}\right)$

(b) Find the centre of curvature of the curve $xy = c^2$ at point (c, c).

5. (a) If

$$x = \cos t + t \sin t;$$
$$y = \sin t - t \cos t,$$

find $\dfrac{d^2y}{dx^2}$.

(b) If $y^{1/m} + y^{-1/m} = 2x$,

prove that

$$(x^2 - 1)y_{n+2} + (2n+1)\, y_{n+1} + (n^2 - m^2)\, y_n = 0.$$

6. (a) State and prove Rolle's theorem.

(b) if f(x) is a quadratic expression, show that the parameter θ in the Lagrange's mean value theorem is ½.

7. (a) Determine the maxima and minima of

$$\frac{1 - x + x^2}{1 + x + x^2}.$$

(b) Prove that the volume of the greatest right circular cone that can be inscribed in a given sphere is 8/27 of the volume of the sphere.

8. (a) Evaluate :

$$\lim_{x \to 0} \frac{\sec \pi x}{\tan 3\pi x}.$$