Register Number :

Name of the Candidate :

5 2 3 2

B.Sc. DEGREE EXAMINATION, 2008

(MATHEMATICS)

(SECOND YEAR)

(PART - III - A - I - MAIN)

(PAPER - II)

640. ANALYSIS - II

(Including Lateral Entry)

December] [Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.
Each question carries TWENTY marks.
(5 × 20 = 100)

1. Evaluate :

(a) $\int \frac{dx}{1 + \tan x}$

(b) $\int \frac{lx + m}{ax + b} dx$

Turn over

(c) $\int \frac{2x - 4}{\sqrt{3x^2 + 4x + 7}}\, dx$

(d) $\int \frac{x^2}{x + 2}\, dx$

$(4 \times 5 = 20)$

2. (a) Prove that

$$\int_a^b f(x)\, dx = \int_a^b (a + b - x)\, dx$$

and hence evaluate

$$\int_{\pi / x}^{(n-1)\,\pi/n} x \sin^3 x \, dx$$

(b) Prove that

$$\int_0^{\pi} \log (1 + \cos x)\, dx = \pi \log (1/2)$$

(12 + 8)

(b) Solve :

$$z = px + qy + 2\sqrt{pq}$$

(c) Solve :

$$(x^2 - yz)\, p + (p^2 - zx)\, q = z^2 - xy$$

(6 + 6 + 8)

(c) Solve :

$$(D^2 + 16)\, y = 2\, e^{-3x} + \cos 4x$$

(6 + 6+ 8)

9. (a) Solve :

$$(D^2 + 4D + 5)\, y = e^x + x^3 + \cos 2x$$

(b) Solve :

$$(1 - x)\, p + (2 - y)\, q = 3 - z$$

10 (a) Solve :

$$p\,(1 + q^2) = q\,(z - 1),$$

where

$$p = \frac{\partial z}{\partial x} \; ;$$

$$q = \frac{\partial z}{\partial y}$$

3. (a) If

$$U_n = \int_0^a x^n\, e^{-x}\, dx,$$

prove that

$$U_n - (n + a)\, U_{n-1} + a(n-1)\, U_{n-2} = 0$$

(b) Find a reduction formula for

$$\int \frac{x^m}{(\log x)^x}\, dx$$

(10 + 10)

4. (a) Prove that every continuous function is integrable.

(b) If f (x) is integrable in (a, b), prove that | f (x) | is integrable in (a, b).

(10 + 10)

5. (a) Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Turn over

(b) Find the centroid of the catenary

$$y = c \cosh \frac{x}{e}$$

from the origin to the point (x, y).

(10 + 10)

6. (a) Find the moment of inertia of a hollow sphere about a diameter.

(b) Evaluate

$$\iiint \frac{dx\ dy\ dz}{(x + y + z + 1)^3}$$

taken over by the volume bounded by the planes

$x = 0,$

$y = 0,$

$z = 0,$

$x + y + z = 1$ (10 + 10)

7. (a) Solve :

$$\frac{dy}{dx} = \frac{y + 2}{x - 1}$$

(b) Solve :

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{3x^2 + y^2}$$

(c) Solve :

$$x^2 y \frac{dy}{dx} = (x + 1) \sec y$$

(7 + 7 + 6)

8. (a) Solve

$$(x^2 + y^2)(xdx + ydy) = a^2 (xdy - ydx)$$

(b) Solve :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

Turn over