(b) If

$$A = \begin{bmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{bmatrix},$$

find A^{-1}.

6. (a) Find the rank of the matrix

$$\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 7 & 8 & 9 & 10 \\ 7 & 8 & 9 & 1 & 2 \end{bmatrix}.$$

(b) If

$$f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

prove that

$$f(x + y) = f(x) \cdot f(y).$$

Register Number :

Name of the Candidate :

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B.Sc. DEGREE EXAMINATION, 2008

(MATHEMATICS)

(THIRD YEAR)

(PART - III - A - MAIN)

(PAPER - IV)

710. VECTOR CALCULUS AND LINEAR ALGEBRA

(Including Lateral Entry)

December] [Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.
All questions carry equal marks.

(5 × 20 = 100)

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1. (a) Evaluate

$$\int_c f \cdot dr$$

where

$$f = (\sin y)\,\bar{i} + x(1 + \cos y)\,\bar{j}$$

and c is the circle

$$x^2 + y^2 = 1$$

in the xy - plane.

(b) Prove that

(i) $\Delta(f(r)) = [f'(r)]\, \bar{r} / r$

and (ii) $\Delta(r^n) = n\, r^{n-2}\, \bar{r}.$

2. (a) Show that $(r^n \bar{r})$ is solenoidal if $n = -3$.

(b) Prove that

curl $(\phi \bar{F}) = (\text{grad}\,\phi) \times \bar{F} + \phi$ (curl F).

3. Verify Stoke's theorem for

$$\bar{f}\,(x^2 - y^2)\,\bar{i} + 2xy\,\bar{j}$$

over the box bounded by the planes

$$x = 0,\; x = a,\; y = 0,\; y = b,\; z = 0,\; z = c$$

if the face $z = c$ is cut.

4. (a) Prove that

$$\begin{vmatrix} 1 & ab & c\,(a+b) \\ 1 & bc & a\,(b+c) \\ 1 & ca & b\,(c+a) \end{vmatrix} = 0.$$

(b) Using determinants, solve the equations

$$x + y + 2z = 4$$

$$2x - y + 3z = 9$$

$$3x - y - z = 2.$$

5. (a) Prove that any Hermitian matrix can be uniquely expressed as A + iB where A is a real symmetric and B is real skew symmetric.

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9. (a) Prove that any two bases of a finite dimensional vector space V have the same number of elements.

(b) Prove that any vector space of dimension n over a field F is isomorphic to

$$V_n(F) = F^n.$$

10. (a) Let V be a finite dimensional vector space over a field F. Let W be a subspace of V. Then, prove that

(i) $\dim W \leq \dim V$

and (ii) $\dim\left(\frac{V}{W}\right) = \dim V - \dim W.$

(b) Let V be a vector space over F. Let

$$S = \{ V_1, V_2, \ldots, V_n \} \subset V$$

if and only if, S is a maximal linearly independent set.

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7. (a) Find the eigen value and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

(b) Test for consistency the following system of equations. If it is consistent, find the solution :

$$x - 4y - 3z = -16$$

$$4x - y + 6z = 16$$

$$2x + 7y + 12z = 48$$

$$5x - 5y + 3z = 0.$$

8. (a) Show that

$$\{ (1, 1, 0), (1, 0, 1), (0, 1, 1), \}$$

is a basis for R^3 over R.

(b) If A and B are subspaces of a vector space V, prove that $A + B$ and $A \cap B$ are subspaces of V.

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