

9. (a) Prove that

$$\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}.$$

(b) Evaluate :

$$\int_0^{\pi/2} \cos^8 \theta \, d\theta.$$

10. (a) Evaluate :

$$\int_0^1 \frac{dx}{\sqrt{1-x^4}}.$$

(b) Prove that

$$\sqrt{n + \frac{1}{2}} = \frac{(2n)! \sqrt{\pi}}{n! 4^n},$$

for $n = 0, 1, 2, \dots$

Register Number :

Name of the Candidate :

5 2 3 8

B.Sc. DEGREE EXAMINATION, 2008

(MATHEMATICS)

(THIRD YEAR)

(PART - III - A - MAIN)

(PAPER - VI)

730. ANALYSIS - III

(Including Lateral Entry)

December]

[Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.

All questions carry equal marks.

(5 × 20 = 100)

1. Find a Fourier series expansion for the function

$$f(x) = x - x^2 \quad \text{in } -\pi \leq x \leq \pi.$$

Turn over

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2. Find a Fourier series for

$$f(x) = \begin{cases} 1 + x & ; \quad 0 < x < \pi \\ -1 + x & ; \quad -\pi < x < 0 \end{cases}$$

in the range $-\pi$ to π .

3. If

$$f(x) = \begin{cases} x & \text{when } 0 < x < \frac{\pi}{2} \\ \pi - x & \text{when } x > \frac{\pi}{2} \end{cases}$$

expand $f(x)$ as a sine series in the interval $(0, \pi)$.

4. Find the Laplace transform of the following :

(a) $\cos^2 3t$.

(b) $\frac{\sin at}{t}$.

(c) Evaluate :

$$\int_0^{\infty} \frac{e^{-3t} - e^{-6t}}{t} dt.$$

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5. Find the inverse Laplace transform :

(a) $L^{-1} \left(\frac{s}{(s+1)^2 + 9} \right)$

(b) $L^{-1} \left(\frac{4s+5}{(s+1)^2(x+2)} \right)$

6. Using Laplace transform, solve the differential equation

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t,$$

given that $y(0) = y'(0) = 0$.

7. (a) Find the Fourier transform of

$$f(x) = \begin{cases} a - x & \text{for } |x| < a \\ 0 & \text{for } |x| > a > 0. \end{cases}$$

(b) Find the Fourier cosine transform of

$$f(x) = \frac{1}{\sqrt{x}}.$$

8. Solve the difference equation

(i) $y_{n+2} - 7y_{n+1} + 10y_n = 12 \cdot 5^n$.

(ii) $y_{n+2} + 5y_{n+1} + 6y_n = 3n^2$.

Turn over