6

(b) Sum to infinity the series

$$1 + \frac{6}{3!} + \frac{11}{5!} + \frac{16}{7!} + \frac{21}{9!} + \dots$$

Register Number :

Name of the Candidate :

1 2 0 2

B.Sc. DEGREE EXAMINATION, 2010

(MATHEMATICS)

(FIRST YEAR)

(PART - III - A - MAIN)

(PAPER - I)

530. ANALYSIS - I

[Time: 3 Hours

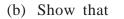
Maximum : 100 Marks

Jo Examicom Answer any FIVE questions. All questions carry equal marks. $(5 \times 20 = 100)$

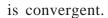
- 1. (a) Prove that $\sqrt{2}$ is irrational.
 - (b) State and prove Dedekind's theorem on real numbers.
- 2. (a) Show that any Cauchy sequence of real numbers is convergent.

Turn over

2



 $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$



3. (a) Find
$$\frac{dy}{dx}$$
, if
(i) $y = 3x^2 e^{3x} \cos x$
(ii) $y = \frac{x(x+1)}{x-1}$
(b) Differentiate
 $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

with respect to

 $\sqrt{1-x^2}$

5

8. (a) Evaluate

$$\lim_{x \to 0} \frac{\log x}{\csc x}$$

(b) Evaluate

$$\lim_{x \to \pi/4} (\tan x)^{\tan 2x}$$
9. (a) Verify Euler's theorem for

$$u = x^3 + y^3 + z^3 + 3xyz.$$
(b) If

$$u = \tan^{-1} \left(\frac{x^2 + y^2}{x + y}\right)$$

Show that

x
$$\frac{\partial u}{\partial x}$$
 + y $\frac{\partial u}{\partial y}$ = $\frac{1}{2}$ sin 2u

10. (a) Sum to infinity the series

$$\frac{2}{6} + \frac{2 \cdot 5}{6 \cdot 12} + \frac{2 \cdot 5 \cdot 8}{6 \cdot 12 \cdot 18} + \dots$$

4

(b) If
$$y = \left(x + \sqrt{x^2 - 1}\right)^m$$

Prove that

$$(1 + x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0.$$

- 6. (a) State and prove Rolle's theorem.
 - (b) If x is positive, prove that

 $1 \ + \ x \ < \ e^x \ < \ 1 \ + \ x \ e^x$

7. (a) Find the maxima and minima of the function

 $x^3 - 18x^2 + 96x + 4$

(b) From a given circular sheet of metal, it is required to cut out a sector so that the remainder can be formed into a conical vessel of maximum capacity. Prove that the angle of the sector removed must be above 66°.

3

4. (a) Find the equation of the tangent to the curve $y = x^3$ at the point

$$\left(\frac{1}{2}, \frac{1}{8}\right)$$

(b) Show that the radius of curvature at any To Exam. Col. point of the cycloid

 $x = a (\theta + \sin \theta)$ and $y = a (1 - \cos \theta)$

is 4a cos
$$\frac{\theta}{2}$$

5. (a) If

 $x = \cos t + t \sin t$

 $y = \sin t - t \cos t$,

find $\frac{d^2 y}{dx^2}$.