(b) Sum to infinity the series

$$1 + \frac{6}{3!} + \frac{11}{5!} + \frac{16}{7!} + \frac{21}{9!} + \ldots..$$

Register Number :

Name of the Candidate :

1 2 0 2

B.Sc. DEGREE EXAMINATION, 2010

(MATHEMATICS)

(FIRST YEAR)

(PART - III - A - MAIN)

(PAPER - I)

530. ANALYSIS - I

May] [Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.
All questions carry equal marks.

(5 × 20 = 100)

1. (a) Prove that $\sqrt{2}$ is irrational.

   (b) State and prove Dedekind's theorem on real numbers.

2. (a) Show that any Cauchy sequence of real numbers is convergent.

Turn over

(b) Show that

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots..$$

is convergent.

3. (a) Find $\frac{dy}{dx}$, if

(i) $y = 3x^2 e^{3x} \cos x$

(ii) $y = \frac{x(x+1)}{x-1}$

(b) Differentiate

$$\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$$

with respect to

$$\sqrt{1 - x^2}$$

8. (a) Evaluate

$$\lim_{x \to 0} \frac{\log x}{\operatorname{cosec} x}$$

(b) Evaluate

$$\lim_{x \to \pi/4} (\tan x)^{\tan 2x}$$

9. (a) Verify Euler's theorem for

$$u = x^3 + y^3 + z^3 + 3xyz.$$

(b) If

$$u = \tan^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$$

Show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$$

10. (a) Sum to infinity the series

$$\frac{2}{6} + \frac{2\cdot 5}{6\cdot 12} + \frac{2\cdot 5\cdot 8}{6\cdot 12\cdot 18} + \ldots..$$

4. (a) Find the equation of the tangent to the curve $y = x^3$ at the point

$$\left(\frac{1}{2}, \frac{1}{8}\right)$$

(b) Show that the radius of curvature at any point of the cycloid

$$x = a\,(\theta + \sin\theta)$$

and $y = a\,(1 - \cos\theta)$

is $4a \cos \dfrac{\theta}{2}$

5. (a) If

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t,$$

find $\dfrac{d^2 y}{dx^2}$.

Turn over

(b) If

$$y = \left(x + \sqrt{x^2 - 1}\right)^m$$

Prove that

$$(1 + x^2)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0.$$

6. (a) State and prove Rolle's theorem.

(b) If x is positive, prove that

$$1 + x < e^x < 1 + x\,e^x$$

7. (a) Find the maxima and minima of the function

$$x^3 - 18x^2 + 96x + 4$$

(b) From a given circular sheet of metal, it is required to cut out a sector so that the remainder can be formed into a conical vessel of maximum capacity. Prove that the angle of the sector removed must be above 66º.