

4

(b) Evaluate  $(\sin x)^{\tan x}$

9. (a) Verify Euler's theorem for

$$u = x^3 + y^3 + z^3 + 3xyz$$

(b) If  $u = (x - y)^2 + (y - z)^2 + (z - x)^2$ ,

prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

10. (a) Sum to infinity the series

$$1 + \frac{9}{8} + \frac{9 \cdot 15}{8 \cdot 16} + \frac{9 \cdot 15 \cdot 21}{8 \cdot 14 \cdot 24} + \dots$$

(b) Sum the series

$$1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots$$

Lim  $x \rightarrow \frac{\pi}{2}$

Register Number :

Name of the Candidate :

**1 2 4 5**

**B.Sc. DEGREE EXAMINATION, 2011**

(MATHEMATICS)

(FIRST YEAR)

(PART - III)

(GROUP - A - MAIN)

(PAPER - I)

**530. ANALYSIS - I**

May ]

[ Time : 3 Hours

Maximum : 100 Marks

*Answer any FIVE questions.*

*All questions carry equal marks.*

(5 × 20 = 100)

1. (a) Prove that any non-empty set of real numbers which is bounded above has a supremum.

(b) Prove that  $\sqrt{3}$  is irrational.

**Turn Over**

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2. (a) Prove that any Cauchy sequence of real numbers is convergent.

(b) Discuss the convergence of the series.

$$\sum \frac{1}{n^k}$$

3. (a) Find  $\frac{dy}{dx}$ , if

(i)  $y = \sin^3(x^2)$

(ii)  $y = \frac{x+4}{x-2}$

(b) Differentiate  $\sec^{-1}x$

with respect to  $x$

4. (a) Find the equation of the tangent to the curve  $y = \frac{6x}{x^2 - 1}$  at the point (2, 4)

(b) Find the radius of curvature of the curve  $y^2 = 4ax$  at the point (2, 0).

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5. (a) Find  $y_n$  if  $y = \frac{1}{1+x^2}$

(b) If  $y = (x^2 + 1)^m$ , prove that

$$(1 + x^2) \frac{dy}{dx} + x y = m^2 y$$

6. (a) State and prove Rolle's theorem.

(b) If  $x$  is positive, show that

$$x - \frac{1}{2}x^2 < \log(1+x) < x$$

7. (a) Find the maxima and minima of the function  $x^3 + 3x^2 - 24x + 20$ .

(b) Prove that the volume of the greatest right circular cone that can be inscribed in a given sphere is  $\frac{8}{27}$  of the volume of the sphere.

8. (a) Evaluate :  $\int_0^1 \frac{1}{1+x^2} dx$

**Turn Over**