(b) State and prove Bernstein theorem. (10)

14. (a) State and prove Littlewood's third principle. (10)

(b) Prove that every absolutely continuous function is the indefinite integral of its derivative. (10)

15. (a) If $a_n \geq 0 \ \forall_n$, prove that $\pi\,(1-a_n)$ converges iff the series $\sum a_n$ converges. (10)

(b) State and prove Cauchy condition for infinite product. (10)

Register Number :

Name of the Candidate :

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M.Sc. DEGREE EXAMINATION, 2007

(MATHEMATICS)

(FIRST YEAR)

(PAPER - II)

120. REAL ANALYSIS

(Revised Regulations)

May] [Time : 3 Hours

Maximum : 100 Marks

PART – A (8 × 5 = 40)

Answer any EIGHT questions.
All questions carry equal marks.

1. State and prove generalized Mean – Value theorem.

2. If f and g are of functions of bounded variation on [a, b], prove that $f + g$ is of bounded variation.

Turn over

3. If P' is a refinement of P, Prove that

$$U(P', f, \alpha) \leq U(p, f, \alpha)$$

4. If $f \in \Re(\alpha)$, prove that for each $t > 0$ there exists a partition P such that

$$U(P, f, \alpha) - L(P, f, \alpha) < t.$$

5. Give an example to show that a sequence of continuous functions need not converge to a continuous function.

6. State and prove Weierstrass M-test.

7. Prove that [0, 1] is uncountable.

8. If f is of bounded variation [a, b], prove that

$$f(b) - f(a) = P_a^b - N_a^b.$$

9. If $\prod (1 + a_n)$ converges absolutely, prove that it converges.

10. Find the value of $\prod_{n=2}^{\alpha} (1 - n^{-2})$.

PART – B (3 × 20 = 60)

Answer any THREE questions.
All questions carry equal marks.

11. (a) State and prove chain rule for differentiation. (10)

(b) Prove that f is of bounded variation on [a, b] iff f can be expressed as the difference of two increasing functions. (10)

12. Let α be of bounded variation on [a, b]. Let V(n) be the total variation of α on [a, x] and let V(a) = 0. If $f \in \Re(\alpha)$, prove that $f \in \Re(V)$? (20)

13. (a) If $f_n \in \Re(\alpha)$ for each n, if $f_n \to f$ uniformly on [a, b] and if $g_n(n) = \int_a^x f_n(t)\, d\alpha(t)$,

prove that $f \in \Re(\alpha)$ and $g_n \to g$ uniformly where $g(n) = \int_a^x f(t)\, d\alpha(t)$ (10)

Turn over