(b) Let V be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

14. (a) Prove that a polynomial of degree *n* over a field can have at most n roots in any extension field.

(b) If $T, S \in A(v)$ and if S is regular, prove that T and $S T S^{-1}$ have the same minimal polynominal.

15. (a) If N is normal and AN = NA, prove that $AN^* = N^* A$, where A is any linear transformation on V.

(b) Prove that any two finite fields having the same number of elements are isomorphic.

Register Number :

Name of the Candidate :

7700

M.Sc. DEGREE EXAMINATION, 2007

(MATHEMATICS)

(FIRST YEAR)

(PAPER - I)

110. ALGEBRA

(Revised Regulations)

May] [Time : 3 Hours

Maximum : 100 Marks

PART – A ($8 \times 5 = 40$)

Answer any EIGHT questions.
Each questions carries FIVE marks.

1. Let H, K be two subgroups of a group G. Prove that H K is a subgroup of G if and only if, H K = K H.

2. Let G be a group. Let A(G) be the set of all automorphisms of G. Prove that A(G) is also a group.

Turn over

3. Prove that a finite integral domain is a field.

4. Prove that a Euclidean ring possesses a unit element.

5. If $v_1, v_2, \ldots. v_n \in V$ are linearly independent, prove that every victor in their linear span has a unique representation in the form

$\lambda_1 v_1 + \lambda_2 v_2 + \ldots\ldots + \lambda_n v_n$ with the $\lambda_n \in f$.

6. State and prove Schwarz inequality.

7. Let F, K, L be fields. If L is an algebraic extension of K and if K is an algebraic extension of F, prove that L is an algebraic extension of F.

8. Let V be a finite dimensional vector space over a field F. Prove that If $T \in A(V)$ is singular if and only if there exists a non – zero vector v in V such that (v)T = 0.

9. If $T \in A(V)$ is Hermitian, prove that all its characteristic roots are real.

10. If T is unitary and if λ is a characteristic root of T, prove that $|\lambda| = 1$.

PART – B (3 × 20 = 60)

Answer any THREE questions.
Each questions carries TWENTY marks.

11. (a) Let G, $\bar{G}$ be groups Let ϕ be a homomorphism of G onto $\bar{G}$ with Kernel K. Prove that $\frac{G}{K}$ is isomorphic to $\bar{G}$.

(b) If p is a prime number and if p^{α} divides O(G), prove that G has a subgroup of order P^{α}.

12. (a) Let R be a commutative ring with unit element. Let M be an ideal of R. Prove that M is a maximal ideal of R if and only if $\frac{R}{M}$ is a field.

(b) If f(x) and g(x) are primitive polynomials, prove that f(x) g(x) is a primitive polynomial.

13. (a) If V is a finite dimensional vector space over a field F, prove that any two bases of V have the same number of elements.

Turn over