Register Number :

Name of the Candidate :

7 7 0 2

M.Sc. DEGREE EXAMINATION, 2007

(MATHEMATICS)

(FIRST YEAR)

(PAPER - III)

130. DIFFERENTIAL GEOMETRY AND DIFFERENTIAL EQUATIONS

(Revised Regulations)

May] [Time : 3 Hours

Maximum : 100 Marks

SECTION – A (8 × 5 = 40)

Answer any EIGHT questions.
All questions carry equal marks.

1. Define the curvature and torsion of the curve. Find the curvature and torsion of the cubic curve $\vec{r} = (u, u^2, u^3)$.

Turn over

(b) Derive Gauss equations of surface theory.

14. (a) Solve the equation

$$xy'' - (2x-1)y' + (x-1)\, y = e^x.$$

(b) Find the series solution of the equation.

$$2\, x^2\, y'' + x\,(2x+1)y' - y = 0.$$

15. (a) Define gamma function. Show that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

(b) Obtain Rodrigue's formula which gives an expression for $P_n(x)$.

2. If the radius of spherical curvature is constant, prove that the curve either lies on a sphere or has constant curvature.

3. Prove that the curves of the family $\frac{v^3}{u^2}$ = constant are geodesics on a surface with metric

$v^2\, du^2 - 2\, u\, v\, du\, dv + 2u^2\, dv^2$; $(u > 0, v > 0)$.

4. State and prove the normal property of geodesics.

5. Find the Gaussian curvature of the surface

$$x = u + v, \quad y = u - v, \quad z = uv.$$

6. If there is a surface of minimum area passing through closed space curve, show that it is necessarily a minimal surface.

7. Solve the equation $y'' + 4y = 4\tan 2x$ using variation of parameters method.

8. Find the general solution of

$$2x^2\, y'' + x\,(2x + 1)y' - y = 0.$$

9. Locate and classify the singular points of

$$(3x + 1)\, xy'' - (x + 1)y' + 2y = 0.$$

10. Prove the following :

(i) $\frac{d}{dx}\,(x^p\, J_p(x)) = x^p\, J_{p-1}(x)$

(ii) $J_{1/2}\,(x) = \sqrt{\frac{2}{\pi x}}\,\sin x.$

SECTION – B $(3 \times 20 = 60)$

Answer any THREE questions.
All questions carry equal marks.

11. (a) Find the arc length of the curve given as the intersection of the surfaces

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad x = a \cosh\left(\frac{z}{a}\right)$$

(b) State and prove the fundamental existence theorem for space curves.

12. (a) Obtain Liouville's formula for K_g.

(b) State and prove Tissot's theorem.

13. (a) Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature is zero.

Turn over