(b) Prove that :

$$\frac{d}{dx}[x^p J_p(x)] = x^p J_{p-1}(x)$$

and $$\frac{d}{dx}[x^{-p} J_p(x)] = -x -p J_{p+1}(x).$$

13. (a) Show that the equation :

$$x_p - y_q = x$$

and $$x^2 p + q = xz$$

are compatible and find their solution.

(b) Using Jacobi's method, find complete integral of the equation $p^2 x + q^2 y = z$.

14. (a) Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y.$$

(b) Obtain the solution, valid when $x, y > 0, xy > 1$ of the equation

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x+y} \text{ such that}$$

Register Number :

Name of the Candidate :

7691

M.Sc. DEGREE EXAMINATION, 2007

(MATHEMATICS)

(FIRST YEAR)

(PAPER - IV)

540. DIFFERENTIAL EQUATIONS WITH APPLICATIONS

(New Regulations)

May] [Time : 3 Hours

Maximum : 100 Marks

PART – A (8 × 5 = 40)

Answer any EIGHT questions.
All questions carry equal marks.

1. Show that $y = C_1 e^{2x} + C_2 x e^{2x}$ is the general solution of $y'' - 4y' + 4y = 0$ on any interval. Find also the particular solution for which $y(0) = 2$ and $y'(0) = 2$.

Turn over

2. Find the general solution of $y'' + xy' + y = 0$ in terms of power series in x.

3. Prove that :

$$\frac{d}{dx}(J_o(x)) = -J_1(x)$$

and $$\frac{d}{dx}(xJ_1(x)) = xJ_o(x).$$

4. Using Gamma function, prove that :

$$(n + ½)! = \frac{(2n+1)!}{2^{2n+1} . n!}\sqrt{\pi}$$

5. Find the general integrals of the partial differential equation $Z(xp - yq) = y^2 - x^2$.

6. Find the equation of the system of surfaces which cut orthogonally the cones of the system

$$x^2 + y^2 + z^2 = cxy.$$

7. If $u = f(x + iy) + g(x - iy)$ where f and g are arbitrary, show that

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0.$$

8. Find particular integral of $(D^2 - D')Z = 2y - x^2$.

9. A gas is contained in a regid sphere of radius a. Show that if C is the velocity of sound in the gas, the frequencies of purely radical oscillations are

$$\frac{c\,z_i}{a} \text{ when } z_1, z_2 \,.....$$

are positive roots of the equation $\tan z = z$.

10. Derive the D′Ahembert's solution of the one dimensional wave equation.

PART – B (3 × 20 = 60)

Answer any THREE questions.
Each question carries equal marks.

11. (a) Find the particular solution of $y''+y = \sec x$.

(b) For the differential equation

$$2x^2y'' + x(2x+1)y' - y = 0.$$

locate and classify its singular points in x axis.

12. (a) Find the general solution of the differential equation

$$x(1-x)y'' + \left(\frac{3}{2} - 2x\right) y'+2y = 0$$

near the singular point $x = 0$.

$$z = 0,\ p = \frac{2y}{x + y}$$

on the hyperbola xy = 1.

15. If the string is released from rest in the position y = f(x), show that the total energy of the string is

$$\frac{\pi^2 T}{4\,l}\ \Sigma\ s^2 {k_s}^2$$

where

$$k_s = \frac{2}{l} \int_0^1 f(x) \sin\left(\frac{s\,\pi\,x}{l}\right) dx$$

The mid-point of the string is pulled aside through a small distance and then released. Show that in the subsequent motion the fundamental mode contributes $\frac{8}{\pi^2}$ of the total energy.

5

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