Ex/BESUS/ME-607/06

B.E. (ME) Part-III 6th Semester Examination, 2006

Numerical Methods and Computer Programming

(ME-607)

Time : 3 hours **Full Marks : 100**

Use separate answerscript for each half.
Answer SIX questions, taking THREE from each half.
The questions are of equal value.
Answers should be brief and to the point.

FIRST HALF

1. Solve the following set of 4 linear algebraic equations by the LU-Decomposition (Doolittle's) method with partial pivoting.

$$x_1 + 2x_2 - 3x_3 + 5x_4 = 1$$
$$x_1 + x_2 - 2x_3 + 7x_4 = 4$$
$$2x_1 + 7x_2 - 7x_3 + 7x_4 = 2$$
$$4x_1 + 9x_2 - 10x_3 + 14x_4 = 6$$

Employ partial pivoting as and when necessary.

2. The following set of x-y data is obtained in an experiment. The data is to be given a fit by an approximately curve having the form $y = ax^b$ where a, b are constants. Determine the values of a and b such that the approximating function fits the given data in the least square sense giving equal weights to the errors in y.

x	1.0	2.0	3.0	4.0	5.0
y	0.5	2.0	4.5	8.0	12.5

If you use any formula, then prove it.

3. a) In cubic spline interpolation, the cubic polynomial in any i^{th} interval may be expressed as

$$f_i(x) = \frac{y_i''}{6}\left[\frac{(x_{i+1}-x)^3}{h_i} - h_i(x_{i+1}-x)\right] + \frac{y_{i+1}''}{6}\left[\frac{(x-x_i)^3}{h_i} - h_i(x-x_i)\right] + y_i\left(\frac{x_{i+1}-x}{h_i}\right) + y_{i+1}\left(\frac{x-x_i}{h_i}\right)$$

where x_i, y_i = x, y values of any i^{th} data point

h_i = length of the i^{th} interval = $x_{i+1} - x_i$

y_i'' = 2^{nd} derivative of $f_i(x)$ evaluated at x_i

The second derivatives at the interior knots are evaluated from equation set obtained by satisfying first derivative continuity of cubic functions of adjacent intervals at the connecting knot. Show that the equation set to solve for y_2'' through y_{n-1}'' may be expressed as

$$\begin{bmatrix} 2(h_1+h_2) & h_2 & & & \bigcirc \\ h_2 & 2(h_2+h_3) & h_3 & & \\ & \ddots & \ddots & \ddots & \\ & & h_{n-3} & 2(h_{n-3}+h_{n-2}) & h_{n-2} \\ \bigcirc & & & h_{n-2} & 2(h_{n-2}+h_{n-1}) \end{bmatrix} \begin{Bmatrix} y_2'' \\ y_3'' \\ \vdots \\ \vdots \\ y_{n-1}'' \end{Bmatrix} = 6 \begin{Bmatrix} \frac{y_3-y_2}{h_2} - \frac{y_2-y_1}{h_1} \\ \frac{y_4-y_3}{h_3} - \frac{y_3-y_2}{h_2} \\ \vdots \\ \frac{y_n-y_{n-1}}{h_{n-1}} - \frac{y_{n-1}-y_{n-2}}{h_{n-2}} \end{Bmatrix}$$

b) Write a Fortran program to generate and print the coefficient matrix and right hand side vector of the equation set given in problem 3(a), in solving the 2nd derivatives at interior knots. The program should read number of data, values of x, y data interactively.

4. Solve the following second order ODE by decomposing it into two first order ODES.

$$\frac{d^2y}{dx^2} + 0.5\frac{dy}{dx} + 7y = 0 \quad \text{with } y(0) = 4 \text{ and } y'(0) = 0$$

Obtain the dependant variables at x = 1.0 with a step size h of 0.5 employing non-self starting predictor-corrector type modified Euler's method with single correction.

5. a) Define condition index. Where is it used?

b) Distinguish clearly between curve fitting and interpolation.

c) What is a linear form in curve fitting? What is its advantage?

d) What is standard error in curve fitting? How does it help the analyst?

SECOND HALF

6. a) Explain the successive bisection method to compute the roots of the equation of a single variable.

b) Write down a computer programme to implement the bisection method.

7. a) Explain the principle of Graeffe's root squaring method to find the roots of a polynomial equation with real coefficients.

   b) Find all roots of the equation
   $x^3 - 2x^2 - 5x + 6 = 0$, by Graeffe's method.

8. a) Derive Lin-Bairstow's algorithm for finding out a quadratic factor of a polynomial equation.

   b) Outline the procedure for implementing the above algorithm manually in a tabular form.

9. a) What is meant by quadrature formula.

   b) Derive the quadrature formula for numerical integration using a second degree parabola.

   c) Use 2-point Gauss-Legendre quadrature formula to compute the integration $\int_0^{\pi/4} \sin x \, dx$. Find the percentage error in the numerical computation when compared with the exact-value.

10. a) Prove the following :

    i) $\Delta = e^{hD} - 1$

    ii) $\nabla = 1 - e^{-hD}$

    b) Deduce Gregory Newton's forward and backward interpolation formulae.

    c) Write down a computer programme to implement Lagrange interpolation method.

———