. £ <^-y , B.E. 6th Semester Examination, May, 2006

-.» %£y*J? vy^jComputer Science & Technology Department

V\iJJ i^x jjfdAi\eU . Theory of Computer Science (CST - (301)

F.M.: 100 TIME : 3 hrs

- Attempt Question No. 1 and any five from the rest.
- Answers should be in your own words as far as practicable.
- Make assumptions as and when necessary and state them at proper places.

1. Write short notes on any four from the following.

(a) Grammar to compute functions

(b) Godel Number of a string

(c) Representation of languages,

(d)" Class of Regular Languages

(e) Nondeterministic Turing Machine to accept languages

[4x5]

2. (a) Formally define the following concepts.

i. Regular Expressions over an alphabet £ ii.
A Regular Expression represents a language

(b) Let £ = *{a, b}*. Write regular expressions for the following sets:

i. All strings in £* with no more than three a's ii. All strings in £* with exactly one occurrence of the substring *aaa*

(c) Construct a deterministic finite automaton to accept the language *{to £ {a, b}* : ui* has neither *aa* not *bb* as a substring}.

[5+6+5]

3. (a) Construct a non-deterministic finite automaton M = *(K,* £, A, *s, F)* to accept the language represented by *((abUaab)*a*)*.*

(b) Some authors define a nondeterministic finite automaton (NDFA) to be a quintuple *(K,T,,A,S,F),* where *K,* £, A and *F* are denned as usual and 5 C *K* is a finite set of initial states, in the same way that *F* is a finite set of final states. The automaton may nondeterministically begin operating in any of these initial states. Formally define the concept of such an automaton accepts a string *ui.* Prove why this definition of NDFA is not more general than the standard one in any significant way.

(c) Given a nondeterministic finite automaton M = *(K,* £. A, *s, F),* propose an algorithm to compute *E(q),* the empty closure of the state *q.*

[5+6+5]

4. (a) Prove that the language *{UJU : ui* 6 *{a,b}*}* is not regular.

(b) Prove that the class of context free languages is not closed under intersection.

(c) Show that the following languages are context free.

i. $\{a^m b^n : m \wedge n\}$

ii. $\{a^i tPc^k : i, j, k > 0$ and $j = i + k\}$

5. Let M — $(K, \text{E}, F, \text{A}, \text{s}, F)$ be a pushdown automaton. The **language accepted by M by final state** is defined as follows.

$$L_f(M) = \{OJ \in \text{E}^* : (s,u,e) \text{ \-}*_M (/, \text{e}, \text{a}) \text{ for some } f \boldsymbol{e} F, a\ 6\ \text{F}^*\}$$

(a) Show that there is a pushdown automaton M' such that $L(M') — Lf(M)$.

(b) Show that there is a pushdown automaton M'' such that $Lf(M'') = L(M)$.

[8+8]

6. (a) Let G = (V, S, i?, 5") be a context free grammar. Formally define the concept of the **derivation** of UJ in G, u 6 E*.

(b) Formally define the concept of the **leftmost derivation** of a; in G.

(c) Prove that for every **derivation** of uj in G there exists an equivalent **leftmost derivation** of LU in G.

(d) Construct a pushdown automaton $M = (K, \text{E}, F, \text{A}, s, F)$ to accept the language $\{a^m b^n : m > n\}$.

[3+3+5+5]

7. (a) Formally define the concept of the function / : $J\backslash f^k$ —> N being Turing Computable.

(b) Prove that every Turing computable function from strings to strings, or numbers to numbers, is gram matically computable.

(c) Construct a grammar G = (V, S, R, S) that generates the language $\{a^n b^n c^n : \text{n} > 0\}$.

[4+8+4]

8. (a) Formally define the concept of a language L being Turing decidable.

(b) *Nondeterministic Turing machine is used to accept languages and not to decide languages* - justify.

(c) Construct a nondeterministic Turing machine $M = (K, \text{E}, \text{A}, 5)$ that accepts the language represented by $a^*abb^*baa^*$.

(d) Construct a Turing machine $M = (K, \text{E}, S, s)$ that computes the function $i\backslash\backslash : M^A$ —> M such that 7r|(m, n_2,n_3,n_4) = n_2.

[2+3+5+6]

9. Show from definition that the following functions are primitive recursive.

(a) $K] : M^k\text{-},N, \quad k,j > 0, K\}(_{ni},n_2, \ldots, n_k) = j$

(b) / : $J\backslash f^4$ —> Af, $f(ni, 722, \text{"3}, \text{^4}) = 9(2, ri2, n\backslash,n\backslash), g$ is primitive recursive

(c) $sg : M$ —> {0,1}, $sg(n)$ = 1 if n = 0 and $sg(n)$ = 1 otherwise

(d) $It : Af^2$ —> {0,1}, $It(ni,ri2)$ — 1 if $n\backslash < n_2$ and $lt\{n\backslash,ni)$ — 0 otherwise

[4x4]