Ex/BESUS/ CST-401/06

B.E. (CST) Part-II 4th Semester Examination, 2006

Discrete Structures

(CST-401)

Time : 3 hours **Full Marks : 100**

Use separate answerscript for each half.
Answer SIX questions, taking THREE from each half.
***Two** marks are reserved for neatness in each half.*

FIRST HALF

1. a) State Handshaking lemma. Use it to prove that every planar graph has at least one vertex of degree < 5.

   b) Prove that K_{33} is non-planar.

   c) Every planar graph can be embedded on the surface of a plane in which some specified face in exterior - Justify this fact. (7+6+3)

2. a) Define outer planar graph. Prove that vertex connectivity of an outer planar graph is two. (Assume the graph is biconnected).

   b) Edge connectivity of a planar graph is at most 5. Justify.

   c) Define 1-isomorphism. Prove that rank of a graph is invariant under 1-isomorphism. (6+5+5)

3. a) Prove that every planar graph is 5-colorable.

   b) Define chromatic polynomial of a graph. Derive chromatic polynomial of a tree of n vertices.

   c) Derive a generating function for the numeric function (1, 2, 3, ..., r, ...).

   (6+5+5)

4. a) Solve the following recurrence relation

   $$4a_r-20a_{r\_}i + 17a_{r\_2} - 4a_{r\_3} = 0.$$

   b) Find particular solution of

   $$a_r- 2a_{r\_}! - 3.2^r.$$

   c) Use suitable generating function to solve the recurrence relation given below.

   $$a_r = 3\,a_{r\_}, + 2\ ,\ r > 1\ ,\ a_0 = 1.$$ (6+4+6)

SECOND HALF

5. a) Define : Logical consequence.

   b) Given : If it is Sunday and nice weather then we go swimming. Today is Sunday. Weather is nice.

   Show that "we will go swimming" is logical consequence of the above text.

   c) What is logical paradox? Give an example. (3+8+5)

6. Obtain the clause form of the following. (4x4)

   i) All that glitters is not gold.

   ii) Only one person spoke at the meeting.

   iii) There is no business like showbusiness.

   iv) 5 is a prime number and it is odd, therefore there exists an odd prime number.

7. a) Write down the converse of the following statement about integers :

   Ifx *andy* are odd, thenx-^ is even.

   Is the statement you wrote down true or false?

   Prove or disprove your answer.

   b) Prove the following statements, where m and n are integers.

   If $x = 5m + 6$ *and* $y=5n + 6$, then $xy= 5k+6$ for some integer k.

   c) Prove the following statement about divisibility of integers

   If $d\backslash a$ and $d\backslash b$ then $d\backslash(ax + by)$ for any integers x *andy*.

   d) Prove the following iff statement about integers.

   x is odd if and only if 81 $(x^2 - 1)$. (4x4)

8. a) What is Principle of Mathematical Induction?

   b) Prove by principle of mathematical induction that the following function computes $2 + 4 + ... +2n$ for any natural number n :

   f(n) = if $n = 0$ then 0 else $f(n\text{-}l) + 2n$

   c) Use Induction to prove that each function performs the stated task,

   i) The function g computes the number of nodes in a binary tree :

   g(D= if r = ( ) t h e n O

   else 1 + g (left (T)) + g (right (7)).

   ii) The function h computes the number of leaves in a binary tree :

   $h(T)$= if r = (> t h e n O

   else if T = tree ((>, x, < » then 1

   else h (left (T)) + h (right (T)). (1+5+10)