Ex/BESUS/M-501/07

B.E. (CST) Part-III 5th Semester Examination, 2007

Mathematics-V

(M-501)

Time : 3 hours **Full Marks : 100**

Use separate answerscript for each half.

FIRST HALF

(*Answer any FIVE questions.*)

1. a) Show that the square of any integer is of the form 4n or 4n+1 for some integer n.

   b) If $ac \equiv bc \pmod{m}$ and gcd (c, m) = d, then prove that $a \equiv b \left(\bmod \frac{m}{d}\right)$.

   c) For any two integers a and b, $a \equiv b \pmod{m}$ if and only if a and b leave the same remainder when divided by m. **[3+3+4]**

2. a) State and prove Fermat's Theorem. Using Fermat's Theorem show that if p be a prime and a is any integer then

   $$a^p \equiv a \pmod{p}$$

   b) Show that $2^{41} \equiv 3 \pmod{23}$. **[7+3]**

3. a) Show that the number of pendant vertices in a binary tree is $(n+1)/2$, where n is the number of vertices in the tree.

   b) Prove that the number of internal vertices in a binary tree is one less than the number of pendant vertices.

   c) Prove that the number of vertices in a binary tree is always odd. **[4+3+3]**

4. a) Show that in a simple graph with n number of vertices and k number of components can have maximum $(n-k)(n-k+1)/2$ number of edges.

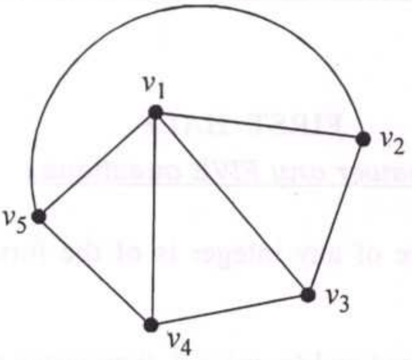
   b) Prove that a circuit free graph with n vertices and $(n-1)$ edges is a tree. **[6+4]**

5. a) Prove that every circuit has an even number of edges in common with any cut-set.

b) Prove that with respect to a given spanning tree T, a chord C_i, that determines a fundamental circuit Γ occurs in every fundamental cutset associated with the branches in Γ and in no other. **[4+6]**

6. a) Prove that every tree with two or more vertices is 2-chromatic.

b) Find the chromatic polynomial of the following graph :



c) Prove that a graph of n vertices is a complete graph if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda-1)(\lambda-2) \,....\, (\lambda-n+1)$. **[4+3+3]**

7. a) Without using truth tables show that

i) $R \rightleftarrows S \Leftrightarrow (R \wedge S) \vee (\neg R \wedge \neg S)$

ii) $\neg (P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q) \Leftrightarrow (\neg P \vee Q)$

b) Obtain a disjunctive normal form of

$P \rightarrow ((P \rightarrow Q) \wedge \neg (\neg Q \vee \neg P))$ **[6+4]**

SECOND HALF

(*Answer Q.No.8 and TWO from the rest.*)

8. a) Find the condition of convergence and order of convergence of the Fixed-point Iterative method.

b) Evaluate the real root of the equation $x^2 = \sin x$ correct to four decimal places by Newton-Raphson method. **[(5+5)+8]**

9. a) Derive Newton's forward interpolation formula with its error term.

b) The population of a town in the decennial census was as given below. Estimate the population for the year 1895 using Lagranges interpolation formula :

Year	: x	1891	1901	1911	1921	1931
Population (in thousand)	: y	46	66	81	93	101

[(5+6)+5]

10. a) Derive Simpson's $1/3$-rd quadrature formula with its error term.

b) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$, correct to 3 decimal places by the Trapezoidal and the Simpson's rules with h = 0.125. **[(5+5)+(3+3)]**

11. a) Derive the second order Runge-Kutta formula and show that the error in this formula is of order h^3.

b) Using the fourth-order R-K method find the value of y(0.2), y(0.4) and y(0.6) when y(0) = 0 and that $\frac{dy}{dx} = x + y$.
Finally find the value of y(0.8) using predictor-corrector method. **[(3+3)+(6+4)]**

12. a) Establish the Finite-difference method for the solution of a following two-point boundary value problem : $y''(x) + f(x)\,y'(x) + g(x)\,y(x) = r(x)$ with boundary conditions $y(x_0) = a$, $y(x_n) = b$.
Using the above method find y(0.5) by considering the equation $y''(x) + y(x) + 1 = 0$, with the boundary conditions y(0) = 0, y(1) = 0. (Taking h = ½)

b) Find the solution, to three decimals of the system

$$83x + 11y - 4z = 95$$
$$7x + 52y + 13z = 104$$
$$3x + 8y + 29z = 71$$

using Gauss-Seidal method. **[(7+3)+6]**

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