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Ex/BESUS/MA-303/07

B.E. (CST) Part-II 3rd Semester Examination, 2007

Mathematics-IIIC

(MA-303)

Time : 3 hours **Full Marks : 70**

Use separate answerscript for each half.

FIRST HALF

(*Answer any THREE Questions.*
Two marks are reserved for general proficiency.)

1. a) State and prove the Cauchy Integral Formula. (A formal proof is needed).

   b) Suppose f(z) is defined by the equations

   $$f(z) = \begin{cases} 1 & \text{when } y < 0 \\ 4y & \text{when } y > 0 \end{cases}$$

   and C is the arc from $z=-1-i$ to $z=1+i$ along the curve $y=x^3$. Evaluate $\int_C f(z)\,dz$. **[6+5]**

2. a) State Dirichlet's conditions.

   b) Obtain the Fourier series generated by the function $f(x)=x^2$, $-\pi \le x \le \pi$. Does this function satisfy Dirichlet's conditions? If so, why? Discuss the convergence of the Fourier series you have obtained.

   c) Prove that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots = \frac{\pi^2}{6}$. **[3+6+2]**

3. a) Integrate $f(z) = e^{iz^2}$ around a suitable contour to evaluate the integrals

   $$\int_0^\infty \cos(x^2)\,dx \text{ and } \int_0^\infty \sin(x^2)\,dx.$$

   b) Use residue calculus to show that

   $$\int_0^{2\pi} \frac{d\theta}{a + b\sin\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}, \quad a > |b|.$$ **[7+4]**

4. a) State and prove Jordon's lemma.

   b) Integrate e^{-z^2} around the rectangle whose vertices are $-R$, R, $R+ia$, $-R+ia$, where a is real and positive. Hence show that

   $$\int_{-\infty}^{\infty} e^{-x^2} \cos(2ax)\,dx = \sqrt{\pi}\, e^{-a^2}$$ **[5+6]**

5. A string of length λ is tied to two fixed points. The string is of uniform tension along its length and has uniform mass per unit length. It is given an initial displacement $y = a\sin^3\left(\frac{\pi x}{\lambda}\right)$ where y is the displacement at a distance x from one end of the string and released from rest. Find the motion of the string. **[11]**

SECOND HALF

(*Answer Q.No.6 and TWO from the rest.*)

6. i) Let a, b, c be integers such that g.c.d. (a, c) = g.c.d. (b, c) = 1. Prove that g.c.d. (ab, c) = 1.

   ii) Let R be a relation on the set A = {1, 2, 3, 4, 5, 6, 7} defined by R = {(a, b) $\in$ A x A : 4 divides $a-b$} then find domain and range of R and R^{-1}.

   iii) Determine which of the mappings $f : R \to R$ are one-one and which are onto R. (a) $f(x) = x + 4$, (b) $f(x) = x^2 \quad \forall x \in R$.

   iv) Write the proof if the following statements are true, otherwise give a counter example.

   a) Every group of four elements is commutative.

   b) Every finite ring with unit element 1 is an integral domain. **[15]**

7. a) Let R = {(a, b) ¦ a, b are rationals and $a-b$ is an integer}
   Prove that R is an equivalence relation on set of rationals.

   b) Let H be a subgroup of G. If $x^2 \in H$ for all $x \in G$ then prove that H is a normal subgroup of G and G/H is abelian.

   c) Let $G = \langle a \rangle$ be a finite cyclic group of order n. Show that a^k is a generator of G iff g.c.d. (k, n) = 1 where k is a positive integer. **[10]**

8. a) Let G be a finite cyclic group of order m. Then prove that for every positive divisor d of m, there exists a unique subgroup of G of order d.

   b) In the ring z_8 and z_6 find the following elements.

   i) the invertible elements

   ii) the nilpotent elements

   iii) the zero divisors.

   c) Prove that a ring R is commutative iff $(a+b)^2 = a^2 + 2ab + b^2$ for any $a, b \in R$. **[10]**

9. a) Examine whether the set of vectors $\{(3, 0, 2), (7, 0, 9), (4, 1, 2)\}$ are linearly independent or not?

b) Show that the set V of all ordered pairs of positive real numbers with operations defined by

$$(x_1, x_2) + (y_1, y_2) = (x_1 y_1, x_2 y_2)$$
$$c(x_1, x_2) = (x_1^c, x_2^c)$$ is a vector space.

c) Show that the set $W = \{(a, 0, b, 0) \mid \text{where } a, b \text{ are reals}\}$ is a subspace of R^4.

[10]

10. a) Prove that intersection of two subspaces of a vectorspace V(F) is always subspace.

b) Show that the number of vertices of odddegree in a graph is always even.

c) What is a simple graph? Give an example.

d) Verify whether the set of all even integers form a field or not. **[10]**

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