Ex/BESUS/MA-303/07

B.E. (CST) Part-II 3rd Semester Examination, 2007

Mathematics-IIIC (MA-303)

Time: 3 hours

Full Marks: 70

Use separate answerscript for each half.

FIRST HALF

(Answer any THREE Questions. Two marks are reserved for general proficiency.)

- 1. a) State and prove the Cauchy Integral Formula. (A formal proof is needed).
 - b) Suppose f(z) is defined by the equations

$$f(z) = \begin{cases} 1 & \text{when } y < 0 \\ 4y & \text{when } y > 0 \end{cases}$$

and C is the arc from z=-1-i to z=1+i along the curve $y=x^3$. Evaluate $\int_C f(z) dz$. [6+5]

- 2. a) State Dirichlet's conditions.
 - b) Obtain the Fourier series generated by the function $f(x) = x^2$, $-\pi \le x \le \pi$. Does this function satisfy Dirichlet's conditions? If so, why? Discuss the convergence of the Fourier series you have obtained.

c) Prove that
$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$
. [3+6+2]

- 3. a) Integrate $f(z) = e^{iz^2}$ around a suitable contour to evaluate the integrals $\int_0^\infty \cos(x^2) dx \text{ and } \int_0^\infty \sin(x^2) dx.$
 - b) Use residue calculus to show that

$$\int_{0}^{2\pi} \frac{d\theta}{a + b \sin \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}, \ a > |b|.$$
 [7+4]

- 4. a) State and prove Jordon's lemma.
 - b) Integrate e^{-z^2} around the rectangle whose vertices are -R, R, R + ia, -R + ia, where a is <u>real</u> and <u>positive</u>. Hence show that

$$\int_{-\infty}^{\infty} e^{-x^2} \cos(2ax) \, dx = \sqrt{\pi} e^{-a^2}$$
 [5+6]

5. A string of length λ is tied to two fixed points. The string is of uniform tension along its length and has uniform mass per unit length. It is given an initial displacement $y = a \sin^3 \left(\frac{\pi x}{\lambda}\right)$ where y is the displacement at a distance x from one end of the string and released from rest. Find the motion of the string. [11]

SECOND HALF (Answer Q.No.6 and TWO from the rest.)

- 6. i) Let a, b, c be integers such that g.c.d. (a, c) = g.c.d. (b, c) = 1. Prove that g.c.d. (ab, c) = 1.
 - ii) Let R be a relation on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ defined by $R = \{(a, b) \in A \times A : 4 \text{ divides } a-b\}$ then find domain and range of R and R^{-1} .
 - iii) Determine which of the mappings $f: R \to R$ are one-one and which are onto R. (a) f(x) = x + 4, (b) $f(x) = x^2 \quad \forall x \in R$.
 - iv) Write the proof if the following statements are true, otherwise give a counter example.
 - a) Every group of four elements is commutative.
 - b) Every finite ring with unit element 1 is an integral domain. [15]
- 7. a) Let R = {(a, b) | a, b are rationals and a-b is an integer}Prove that R is an equivalence relation on set of rationals.
 - b) Let H be a subgroup of G. If $x^2 \in H$ for all $x \in G$ then prove that H is a normal subgroup of G and G/H is abelian.
 - c) Let G = (a) be a finite cyclic group of order n. Show that a^k is a generator of G iff g.c.d. (k, n) = 1 where k is a positive integer. [10]
- 8. a) Let G be a finite cyclic group of order m. Then prove that for every positive divisor d of m, there exists a unique subgroup of G of order d.
 - b) In the ring z_8 and z_6 find the following elements.
 - i) the invertible elements
 - ii) the nilpotent elements
 - iii) the zero divisors.
 - c) Prove that a ring R is commutative iff $(a+b)^2 = a^2 + 2ab + b^2$ for any $a, b \in R$.

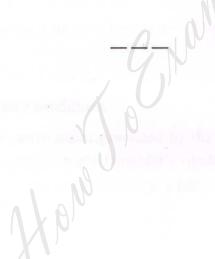
(MA-303)

- 9. a) Examine whether the set of vectors {(3, 0, 2), (7, 0, 9), (4, 1, 2)} are linearly independent or not?
 - b) Show that the set V of all ordered pairs of positive real numbers with operations defined by

$$(x_1, x_2) + (y_1, y_2) = (x_1y_1, x_2y_2)$$

 $c(x_1, x_2) = (x_1^c, x_2^c)$ is a vector space.

- c) Show that the set $W = \{(a, 0, b, 0) \mid \text{where a, b are reals} \}$ is a subspace of \mathbb{R}^4 .
- 10. a) Prove that intersection of two subspaces of a vectorspace V(F) is always subspace.
 - b) Show that the number of vertices of odddegree in a graph is always even.
 - c) What is a simple graph? Give an example.
 - d) Verify whether the set of all even integers form a field or not. [10]



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