

B.E. (CST) Part-II 4th Semester Examination, 2007

Probability and Statistics

(MA-402)

Time : 3 hours

Full Marks : 70

Use separate answerscript for each half.

Answer SIX questions, taking THREE from each half.

Two marks are reserved for general proficiency in each half.

FIRST HALF

1. a) In a certain examination, six papers are set, and to each are assigned 100 marks as a maximum. A student is assumed to get any score out of 100 with equal chance. Show that the probability of his/her getting forty percent of the whole number of marks is

$$\frac{1}{5!} \left\{ \frac{(245)!}{(240)!} - 6 \frac{(144)!}{(139)!} + 15 \frac{(43)!}{(38)!} \right\} \\ (101)^6$$

- b) State and prove Bayes' theorem. [7+4]

2. a) What do you understand by the term "matching problem"? Discuss.
b) If X denotes the number of matches in a "matching problem", find E(X) and Var(X). [4+7]

3. a) Find the moment generating function (m.g.f.) of a binomial random variable with mean np (and variance np(1 - p)). Show that if X ~ Bin (n, p) and Y ~ Bin (m, p) and X and Y are stochastically independent, then X + Y ~ Bin (n+m, p).

- b) Consider the distribution having p.d.f. given by

$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, \quad -\infty < x < \infty.$$

Show that this distribution does not have finite mean. Does the m.g.f. exist for this distribution? Give reasons for your answer. [6+5]

(MA-402)

4. a) A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that takes him to safety after 2 hours of travel. The second door leads to a tunnel that returns him to the mine after 3 hours of travel. The third door leads to a tunnel that returns him to the mine after 5 hours of travel. Assuming that the miner is at all times equally likely to choose any one of the doors, find the m.g.f. of X , the time when the miner reaches safety. (Note that our assumption implies that the miner is stupid!)
- b) Deduce the normal equations for a linear regression model, starting from first principles. [6+5]
5. a) Suppose X_1, X_2, \dots, X_n are independent and identically distributed random variables such $X_i \sim N(\mu, \sigma^2) \forall i = 1, 2, \dots, n$. Show that
- $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
 - $\sum_{i=1}^n (X_i - \bar{X})^2 / \sigma^2$ is chi-square distributed with $(n-1)$ degrees of freedom.
- b) Prove the formula :
- $$\text{Var}(X_1 + \dots + X_n) = \sum_{i=1}^n \text{Var}(X_i) + 2 \sum_{i < j} \text{Cov}(X_i, X_j).$$
- What does this formula reduce to, when the X_i 's are independent random variables? Justify your answer. [6+5]

SECOND HALF

6. a) Calculate mean and median of the following frequency distribution :
- | Class-interval | 0 - 8 | 8 - 16 | 16 - 24 | 24 - 32 | 32 - 40 | 40 - 48 |
|----------------|-------|--------|---------|---------|---------|---------|
| Frequency | 8 | 7 | 16 | 24 | 15 | 7 |
- b) Write short notes on :
- (i) Frequency Polygon, (ii) Skewness. [6+5]
7. a) Define consistent and unbiased estimator.
- Let $\{T_n\}$ be a sequence of estimators such that for all $\theta \in \Theta$,
- $E_\theta(T_n) \rightarrow \theta ; n \rightarrow \infty$
 - and $\text{Var}_\theta(T_n) \rightarrow 0$ as $n \rightarrow \infty$.
- Then show that T_n is a consistent estimator of θ .
 $[\Theta$ denotes the parametric space].